## **ISCO 2016 Conference**

4th International Symposium on Combinatorial Optimization

May 16-18 2016, Vietri sul Mare, Italy

## BOOK OF ABSTRACTS

# **Organizing Institutions**



Department of Mathematics University of Salerno

**University of Salerno** 

Mathematical Methods, Models and Applications Laboratory

**Applied Soft Computing** 

Laboratory for Analysis and Modelling of Decision Support Systems

**Paris Dauphine University** 

**University of Sannio** 

### **ISCO 2016** Committees

### **Conference Chairs**

Raffaele Cerulli (University of Salerno, Italy) Satoru Fujishige (Kyoto University, Japan) Ridha Mahjoub (Paris Dauphine University, France)

#### **Steering Committee**

M. Baïou (LIMOS, CNRS, Blaise Pascal University, Clermont-Ferrand, France)

- P. Fouilhoux (Pierre and Marie Curie University, Paris, France)
- L. Gouveia (University of Lisbon, Portugal)
- N. Maculan (Federal University of Rio de Janeiro, Brasil)
- A. R. Mahjoub (Paris Dauphine University, France)
- V. Paschos (Paris Dauphine University, France)
- G. Rinaldi (IASI, Roma, Italy)

#### **Program Committee**

- E. Amaldi (Polytechnic University of Milan, Italy)
- F. Barahona (IBM, T. J. Watson, New York, USA)
- D. Bienstock (Columbia University, New York, USA)
- F. Carrabs (University of Salerno, Italy)
- L. Escudero (Rey Juan Carlos University, Spain)
- M. Fischetti (University of Padova, Italy)
- R. Fukasawa (University of Waterloo, Canada)

- T. Fukunaga (NII, Tokyo, Japan)
- M. Haouari (ENIT, Tunis, Tunisia)
- H. Hirai (University of Tokyo, Japan)
- N. Garg (Indian Institute of Technology, India)
- M. Gentili (University of Salerno, Italy)
- B. Golden (University of Maryland, USA)
- L. Gourvès (University of Paris Dauphine)
- P. Italiano (University of Tor Vergata, Italy)
- I. Kacem (LCOMS, University of Lorraine, Metz, France)
- V. Kaibel (Otto-von-Guericke University, Magdeburg, Germany)
- N. Kamiyama (Kyushu University, Japan)
- S. Kijima (Kyushu University, Japan)
- Y. Kobayashi (University of Tsukuba, Japan)
- M. Labbé (Free University, Brussels, Belgium)
- G. Laporte (HEC, Montréal, Canada)
- L. Liberti (LIX, Ecole Polytechnique, Paris, France)
- A. Lodi (University of Bologna, Italy)
- M. Lübbecke (Aachen University, Germany)
- C. Mannino (SINTEF ICT, Oslo, Norway)
- F. Margot (Carnegie Mellon University, USA)
- S. Martello (University of Bologna, Italy)
- S.T. McCormick (UBC, Vancouver, Canada)
- Y. Milis (Athens University of Economics and Business, Greece)
- K. Nagano (Future University Hakodate, Japan)
- Y. Okamoto (UEC, Tokyo, Japan)
- G. Oriolo (University of Tor Vergata, Italy)
- N. Perrot (Orange Labs, Paris, France)
- F. Rendl (University of Klagenfurt, Graz, Austria)

J.J. Salazar Gonzalez (University of La Laguna, Tenerife, Spain)
M. Sevaux (University of Southern Brittany, France)
D. Shier (Clemson University, USA)
A. Shioura (Tokyo Institute of Technology, Japan)
M.G. Speranza (University of Brescia, Italy)
K. Takazawa (Hosei University, Tokyo, Japan)
S.Tanigawa (RIMS, Kyoto, Japan)
P. Toth (University of Bologna, Italy)
C. Triki (Sultan Qaboos University, Muscat, Oman)
E. Uchoa (UFF Rio de Janeiro, Brasil)
F. Vanderbeck (University of Bordeaux 1, France)
H. Yaman (Bilkent University, Ankara, Turkey)
P.Y. Yin (National Chi Nan University, Taiwan)
Y. Yoshida (NII, Tokyo, Japan)

#### **Organizing Committee**

Raffaele Cerulli (University of Salerno, Italy) Francesco Carrabs (University of Salerno, Italy) Monica Gentili (University of Salerno, Italy) Andrea Raiconi (University of Salerno, Italy) Carmine Cerrone (University of Salerno, Italy) Ciriaco D'Ambrosio (University of Salerno, Italy) Rosa Pentangelo (University of Salerno, Italy) Selene Silvestri (University of Salerno, Italy) Maurizio Boccia (University of Sannio, Italy)

#### **Additional Reviewers**

Eduardo Álvarez-Miranda **Dimitrios** Letsios Georgios Amanatidis Nourine Lhouari Rafael Andrade Alfredo Marín Giannakos Aristotelis Isabel Mendez-Diaz Lucas Assuncao Ryuhei Miyashiro Rémy Belmonte Flavio K. Miyazawa Walid Ben-Ameur Vahan Mkrtchyan Fatiha Bendali Tobias Mömke Hans-Joachim Böckenhauer Pedro Moura Flavia Bonomo Kazuo Murota Laurent Bulteau Giacomo Nannicini Shaowei Cai Graciela Nasini Jose Neto Katrin Casel Daniele Catanzaro Paolo Nobili Audrey Cerqueus Gizem Ozbaygin Leonardo Conegundes Julia Pahl Denis Cornaz Vasileios-Orestis Papadigenopoulos Alexandre Cunha Arnaud Pêcher Ciriaco D'Ambrosio Artur Pessoa Alban Derrien Christophe Picouleau Gustavo Dias Andrea Raiconi **Daniel** Dias Günther Raidl Yuri Faenza Fabrizio Rossi Marcia Fampa Larry Rudolph Giovanni Felici Ruslan Sadykov Moritz Firsching Florian Sikora Stefano Smriglio **Dimitris** Fotakis Manlio Gaudioso Tasuku Soma **Roland** Grappe Ramanujan Sridharan Shinji Imahori Abhinav Srivastav Maximilian John Georgios Stamoulis Andreas Karrenbauer Noriyoshi Sukegawa Ioannis Katsikarelis Shunji Umetani Yasushi Kawase Sebastián Urrutia Mustafa Kemal André van Renssen Esra Koca Yuan Zhou Shungo Koichi Georgios Zois Michael Lampis

## Contents

Preface	9
Plenary Lectures 10	0
Sessions 1	7
MOA1 - Semidefinite Programming	8
MOA2 - Wireless Sensor Networks	3
MOA3 - Storage / Packing Problems	9
MOA4 - Polyhedral Approaches I	5
MOB1 - Scheduling Problems I	9
MOB2 - Routing Problems I	5
MOB3 - Network Design Problems I	1
MOB4 - Polyhedral Approaches II	6
TUA1 - Stable Set Problems	2
TUA2 - Routing Problems II	7
TUA3 - Network Flow Problems	1
TUA4 - Timetable Problems	5
TUB1 - Scheduling Problems II	1
TUB2 - Traveling Salesman Problem	6
TUB3 - Network Design Problems II	2
TUB4 - Polyhedral Approaches III	6

WEA1 - Graph Partitioning
WEA2 - Game Theory / Stochastic
WEA3 - Clustering / Neuro-Fuzzy
WEA4 - Energy Systems I
WEB1 - Scheduling Problems III
WEB2 - Tree Problems
WEB3 - Approximation Algorithms
WEB4 - Decomposition Approaches
WEC1 - Knapsack Problems
WEC2 - Lot Sizing Problems
WEC3 - Closest String / Matching
WEC4 - Energy Systems II

### Author index

158

### Preface

Dear Participants,

welcome to ISCO 2016, the 4th International Symposium on Combinatorial Optimization!

After the previous editions held in Hammamet, Athens and Lisboa, we are happy to welcome you in Vietri sul Mare, in the wonderful setting of the Amalfi Coast.

ISCO is nowadays a highly anticipated biannual meeting for the combinatorial optimization research community.

We are grateful to all authors who contributed to our high-level scientific program. Overall, we received about 160 submissions from researchers in many different countries. Among them, about 100 full papers were submitted for the LNCS post-conference proceedings book, and nearly 40 of them were accepted. We would like to also thank all PC members, the members of the organizing committee and the external reviewers for their excellent work, within demanding time constraints. Our 28 contributed sessions span many different topics of combinatorial optimization. Together with the 4 invited lectures by internationally renowned researchers such as R. Ravi, A. Letchford, A. Frank and V. Kaibel, we are sure that they will be a source of insights and fruitful discussions for all participants.

We hope you will all enjoy the conference and your stay in Vietri!

Raffaele Cerulli Satoru Fujishige A. Ridha Mahjoub ISCO 2016 Co-Chairs

### **Plenary Lectures**

### Improved Approximations for Graph-TSP in Regular Graphs

#### R. Ravi

Carnegie Mellon University, ravi@andrew.cmu.edu

A tour in a graph is a connected walk that visits every vertex at least once, and returns to the starting vertex. We describe improved approximation results for a tour with the minimum number of edges in regular graphs. En route we illustrate the main ideas used recently in designing improved approximation algorithms for graph TSP.

# New graph optimization problems in $NP \cap co-NP$

#### András Frank

Egerváry Research Group, Eövös University Budapest, frank@cs.elte.hu

We show that the following three problems in graph theory belong to  $NP \cap co-NP$ .

1. Wang and Kleitman (1972) characterized degree-sequences of simple k-connected undirected graphs. We solve the corresponding problem for digraphs.

2. Edmonds (1973) characterized digraphs admitting k disjoint spanning arborescences of given root, and his result could be extended to the case when there is no prescription for the localization of the roots. Here we exhibit a much more general result that characterizes digraphs admitting k disjoint branchings with specified sizes  $\mu_1, \mu_2, \ldots, \mu_k$ .

3. Ryser (1958) solved the maximum term rank problem which consisted of characterizing the row-sums and column-sums of (0, 1)-matrices with termrank at least  $\mu$ , or equivalently, characterize the degree-sequences of simple bipartite graphs with matching number at least  $\mu$ . Recently, it turned out that the maximum term rank problem, though not particularly difficult, is not tractable with network flow or matroid techniques since the weighted version is **NP**-complete. Yet, we found a necessary and sufficient condition for the existence of a simple bipartite graph with matching number at least  $\mu$  such that the degree of each node lies between specified lower and upper bounds.

As a major novelty, we show that these three apparently quite distant problems stem out from one common root: a general theorem on covering a supermodular function by a minimal *simple* digraph. Since the corresponding weighted optimization version includes **NP**-complete problems, the new results are certainly out of the range of classic general frameworks such as the one of submodular flows.

In the talk, I outline first the origin and the history of optimization problems concerning optimal coverings of supermodular functions and exhibit then the new developments giving rise to the characterizations indicated above. Finally, some open problems are sketched that are hopeful to be attacked successfully with the new approach.

The new results are joint work with Kristóf Bérczi.

### Some Hard Combinatorial Optimization Problems from Mobile Wireless Communications

#### Adam N. Letchford

Lancaster University, a.n.letchford@lancaster.ac.uk

In the past decade, a revolution in telecommunications has been taking place. There has been an inexorable trend towards mobile wireless communications, in which there are a large number of *portable devices* (such as smartphones) scattered across a geographical region. Each such region is divided into a number of so-called *cells*. Each cell contains a powerful transmitter called a *base station*. When they wish to send or receive data, the portable devices have to send requests to one or more nearby base stations.

It turns out that mobile wireless communications are a rich source of new and difficult combinatorial optimisation problems. These include strategic problems, such as where and when to locate new base stations, tactical problems, such as how much power to give to each base station, and operational problems, such as how to assign incoming user requests to the available frequency bands.

In this talk, we focus on operational problems associated with so-called *orthogonal frequency-division multiple access* (OFDMA) systems. In these systems, there are a large number of channels available, each of which can be allocated to at most one user. On the other hand, a user can be assigned to more than one channel. The rate at which data is transmitted over a given channel is a nonlinear function of the power allocated to that channel, the bandwidth of the channel, and the noise associated with the channel. So one faces the problem of simultaneously assigning channels to users and allocating the available power to the channels. This leads to several different combinatorial optimization problems, depending on the particular objective in question, the side-constraints imposed, and the time-horizon of interest.

We show that some of these joint channel assignment and power allocation problems can be tackled successfully via mixed-integer linear programming, especially if one uses clever pre-processing tricks, strong cutting planes, and symmetry-breaking techniques. On the other hand, some of the problems still present a formidable challenge.

### Describing Integer Points in Polyhedra

#### Volker Kaibel

Otto-von-Guericke University, Magdeburg, kaibel@ovgu.de

Linear mixed integer models are fundamental in treating combinatorial problems via Mathematical Programming. In this lecture we are going to discuss the question how small such formulations one can obtain for different problems. It turns out that for several problems including, e.g., the traveling salesman problem and the spanning tree problem, the use of additional variables is essential for the design of polynomial sized integer programming formulations. In fact, we prove that their standard exponential size formulations are asymptotically minimal among the formulations based on incidence vectors only. We also treat bounds for general sets of 0/1-points and briefly discuss the question for the role of rationality of coefficients in formulations.

Joint work with Stefan Weltge.

### Sessions

### A coordinate ascent method for solving semidefinite relaxations of non-convex quadratic integer programs

Christoph Buchheim

STechnische Universität Dortmund, christoph.buchheim@tu-dortmund.de

#### Maribel Montenegro

Technische Universität Dortmund, maribel.montenegro@math.tu-dortmund.de

#### Angelika Wiegele

Alpen-Adria-Universität, angelika.wiegele@aau.at

We present a coordinate ascent method for a class of semidefinite programming problems that arise in non-convex quadratic integer optimization problems of the form

$$\begin{array}{ll} \min & x^{\top} \hat{Q} x + \hat{l}^{\top} x + c \\ \text{s.t.} & x \in D_1 \times \cdots \times D_n \end{array}$$

where  $\hat{Q} \in \mathbb{R}^{n \times n}$  is symmetric but not necessarily positive semidefinite,  $\hat{l} \in \mathbb{R}^n$ ,  $c \in \mathbb{R}$ , and  $D_i = \{l_i, \ldots, u_i\} \subseteq \mathbb{Z}$  is finite for all  $i = 1, \ldots, n$ . Buchheim et al. (2013) have studied the general case where each  $D_i$  is an arbitrary closed subset of  $\mathbb{R}$ . The authors have implemented a branch-and-bound approach called Q-MIST. Q-MIST needs to solve a semidefinite program (SDP) at each node of the branch-and-bound tree. In Buchheim et al. (2013), Q-MIST was implemented using an interior point method, namely the CSDP library (Borchers (1999)). It is well-known that interior point algorithms are theoretically efficient to solve SDPs, they are able to solve small to medium size problems with high accuracy, but they are memory and time consuming for large scale instances.

We propose an alternative method to the interior point one, that uses the fact that the SDPs we are solving are characterized by a small total number of active constraints and small rank constraint matrices. We exploit this special structure by solving the dual problem, using a barrier method in combination with a coordinate-wise exact line search. Our approach is motivated by an algorithm proposed by Dong (2014). The main ingredient of our algorithm is the computationally cheap update at each iteration and an easy computation of the exact step size.

Compared to interior point methods, our approach is much faster in obtaining reasonable dual bounds. Moreover, no explicit separation and reoptimization is necessary even if the set of primal constraints is large or even infinite, since in our dual approach this is covered by implicitly considering all constraints when selecting the next coordinate.

#### References

- B. Borchers. CSDP, a C library for semidefinite programming. Optimization Methods and Software, 11(1-4):613-623, 1999.
- [2] C. Buchheim and A. Wiegele. Semidefinite relaxations for non-convex quadratic mixed-integer programming. *Math. Program.*, 141(1-2):435– 452, 2013.
- [3] H. Dong. Relaxing nonconvex quadratic functions by multiple adaptive diagonal perturbations. Working paper, Department of Mathematics, Washington State University, March 2014.

### Polyhedral and semidefinite programming approaches for the quadratic set covering problem

Alexandre Salles Da Cunha

Universidade Federal de Minas Gerais, acunha@dcc.ufmg.br

#### Philippe Mahey

Université Blaise Pascal, philippe.mahey@isima.fr

In this paper, we present polyhedral and semidefinite programming (SDP) approaches for solving the Quadratic Set Covering Problem (QSCP): BC, a Branch-and-cut method based on Linear Programming, CPSDP, an SDP cutting plane algorithm and, RCSDP, an SDP Lagrangian Relax-and-cut approach that strengthens Lagrangian duals with the dynamic dualization of polyhedral constraints within SDP relaxations. CPSDP and RCSDP were not embedded in a branch-and-bound search tree, being QSCP heuristics with performance guarantees. All three algorithms make use of a local search procedure that provides good feasible solutions and lifted optimality cuts for QSCP. BC obtained the best computational results in terms of number of optimality certificates and CPU times. RCSDP was the second best algorithm, solving more instances to proven optimality than CPSDP. Thanks to the lower computational cost involved in solving its SDP relaxations, it often obtained smaller duality gaps than CPSDP, for instances unsolved by both.

### A Novel SDP relaxation for the quadratic assignment problem using cut pseudo bases

#### Maximilian John

Max-Planck-Institute for Informatics, mjohn@mpi-inf.mpg.de

#### Andreas Karrenbauer

Max-Planck-Institute for Informatics, karrenba@mpi-inf.mpg.de

The quadratic assignment problem (QAP) is one of the hardest combinatorial optimization problems. Its range of applications is wide, including facility location, keyboard layout, and various other domains. The key success factor of specialized branch-and-bound frameworks for minimizing QAPs is an efficient implementation of a strong lower bound. In this paper, we propose a lower-bound preserving transformation of a QAP to a different quadratic problem allowing for small and efficiently solvable SDP relaxations. This transformation is self-tightening in a branch-and-bound process. Experimental evaluations show promising results, in particular for instances with a small width in one of the dimensions.

### Diagonally dominant programming in distance geometry

Gustavo Dias CNRS LIX, dias@lix.polytechnique.fr

Leo Liberti

CNRS LIX, Ecole Polytechnique, leoliberti@gmail.com

Distance geometry is a branch of geometry which puts the concept of distance at its core. The fundamental problem of distance geometry asks to find a realization of a finite, but partially specified, metric space in a Euclidean space of given dimension. An associated problem asks the same in a Euclidean space of *any* dimension. Both problems have many applications to science and engineering, and many methods have been proposed to solve them. Unless some structure is known about the structure of the instance, it is notoriously difficult to solve these problems computationally, and most methods either will not scale up to useful sizes, or will be unlikely to identify good solutions. We propose a new heuristic algorithm based on: an semidefinite programming formulation, a diagonally-dominant inner approximation of Ahmadi and Hall's, a randomized-type rank reduction method of Barvinok's, and one call to a local nonlinear programming solver.

### An exact and a heuristic approach for maximizing lifetime in sensor networks with coverage and connectivity constraints

Francesco Carrabs University of Salerno, fcarrabs@unisa.it

Raffaele Cerulli University of Salerno, raffaele@unisa.it

Ciriaco D'Ambrosio University of Salerno, cdambrosio@unisa.it

Andrea Raiconi University of Salerno, araiconi@unisa.it

We want to define a schedule for the activation intervals of the sensors deployed inside a region of interest. At all times the activated sensors must monitor a set of interesting target locations and route the information to a central base station, while maximizing the global amount of time over which the network can be operational. We take into account either complete or partial coverage of the set of targets. To optimally solve the problem, we propose a column generation approach that uses a genetic algorithm to overcome the difficulty of solving the subproblem to optimality in each iteration. We also devise a heuristic by stopping the column generation resolution as soon as the columns found by the genetic algorithm do not improve the incumbent solution. We test the algorithm on benchmark instances and compare with previous approaches, showing that our algorithm is highly competitive in terms of solution quality and computational time.

### Maximization of residual capacities for target tracking in wireless sensor networks

Charly Lersteau

Universitè Bretagne Sud, charly.lersteau@univ-ubs.fr

Marc Sevaux Universitè Bretagne Sud, marc.sevaux@univ-ubs.fr

André Rossi Universitè d'Angers, andre.rossi@univ-angers.fr

Raffaele Cerulli University of Salerno, raffaele@unisa.it

Andrea Raiconi University of Salerno, araiconi@unisa.it

Wireless sensor networks are composed of many sensors able to monitor one or several targets in their sensing range. Sensors are randomly dispatched in a region in which targets are moving. Each sensor has a limited battery lifetime and two states: active or inactive. Thus, a way to save energy is to activate only a subset of the sensors. A sensing activity is the action of activating a sensor for tracking targets during a certain time interval. Given a prevision of the trajectories of the targets, we aim at finding an optimal schedule of sensing activities, covering all the targets at any instant of the mission. The objective is to balance the energy among the sensors to preserve the ability to cover the region in further missions. The method runs in two steps. The first one, called discretization, is to turn the problem input into a scheduling problem instance. The monitored area is partitioned into faces, where a face is a surface covered by the same set of sensors. Then, the trajectories of the targets can be seen as sequences of the traversed faces. The time horizon is also split into time windows, so the problem is transformed into a series of face covering problems. The purpose of the second step is to assign activation time to covers, i.e. subsets of sensors able to cover all the targets during a time window. Since the set of all feasible covers is huge, they

cannot be enumerated in practice. Hence a column generation algorithm is used for that purpose. The mathematical models will be presented during the talk along with numerical results. The master problem is linear, continuous and includes capacity and covering duration constraints. Each time window is associated a pricing problem, solved using GRASP meta-heuristic and an ILP solver. This work is open to many perspectives. In particular, the discretization step provides a way to formulate many target tracking problems, such as a problem of robust scheduling of sensing activities.

### The p-cycle star problem: formulations and cutting-plane methods

**Vinícius Morais** *UFMG*, vwcmorais@gmail.com

Geraldo Robson Mateus UFMG, mateus@dcc.ufmg.br

Bernard Gendron CIRRELT, bernard.gendron@cirrelt.ca

The *p*-Cycle Star Problem (P-CSP) is defined in terms of a connected digraph D = (V, A) with a set V of n + 1 vertices, a set A of m arcs and a fixed positive integer p.  $r \in V$  represents a higher order vertex, the root. Two different sets of nonnegative fixed costs are considered, cycle cost  $\{c_{(i,j)} > 0 : (i,j) \in A\}$  and assignment cost  $\{d_{(i,j)} > 0 : (i,j) \in A\}$ . CSP seeks to cluster the vertices into p disjoint vertex subsets (*clusters*),  $\{K^1, ..., K^p\}$ , each one centered at a *cluster-head* vertex  $v^i \in K^i$ , such that: i)  $V \setminus \{r\} = \bigcup_{i=1}^{p} K^{i}$ ; ii)  $W \subset V$  is a *cluster-head* set, where  $r \in W$ and |W| = p + 1; iii) the root and all p cluster-heads are connected with p+1 arcs forming a simple directed cycle  $(W, A_W)$  of D, where  $A_W =$  $\{(i,j) \in A : i,j \in W\}$ ; iv) every other vertex not in W, is connected by an arc to a cluster-head  $i \in W \setminus \{r\}$ . If we denote by intra-cluster star each subdigraph  $(K^i, A_K^i)$  spanning the vertices within a given cluster  $K^i$ , the objective is to minimize *inter* and *intra-cluster* total cost, given by  $\sum_{(i,j)\in A_W} c_{(i,j)} + \sum_{i=1}^p \sum_{(i,j)\in A_K^i} d_{(i,j)}$ . This problem arises specially in the design of wireless sensor networks, where  $\{i = 1, \ldots, n : i \in V\}$  represents the sensor set and  $r \in V$  the mobile sink that visits each cluster-head to gather sensed information on each *cluster*. We present two mixed integer formulation for the problem, one based on a directed multicommodity flow model and the other based on exponentially many subtour elimination constraints. Additionally, two non-standard cutting-plane method based on those formulations are also implemented. Computational experiments on an instance set extended from the literature involving up to 200 vertices is reported, showing the benefits of our approach.

### The directional sensor coverage problem with continuous orientation

Annabella Astorino ICAR-CNR

#### Manlio Gaudioso

DIMES-Università della Calabria, gaudioso@deis.unical.it

#### Giovanna Miglionico

DIMES-Università della Calabria

We present a mixed integer non linear programming formulation of the Directional Sensors Continuous Coverage Problem (DSCCP), where a given set of targets in a plane are to be covered by a set of sensors whose location is known in advance. Sensors are supposed to be directional, that is characterized by a discrete set of possible radii and aperture angles. Decisions to be made are about orientation (which in our approach can vary continuously), radius and aperture angle of each sensor, taking into account possibility of keeping one or more sensors switched off. The objective is to get minimum cost coverage of all targets. We incorporate into the objective function penalty cost for possibly uncovered targets. We prove NP-hardness of DSCCP and introduce a Lagrangian relaxation model. A dual ascent procedure is also presented. It is based on acting on one multiplier at a time and it is completed by a heuristics to find a feasible solution at each ascent iteration. Finally we report the results of the implementation of the method on a set of test problems.

### A constraint programming model for integrated quay and yard operations at a container terminal

Damla Kizilay

Yasar University, damla.kizilay@yasar.edu.tr

Deniz Türsel Eliiyi

Yasar University, deniz.eliiyi@yasar.edu.tr

This paper examines the integrated problem of quay crane assignment, quay crane scheduling, yard location assignment, container stacking and vehicle dispatching operations at a container terminal. Addressing these problems simultaneously, we focus on practical situations by taking into account the realistic restrictions of a terminal. The main objective of the problem is stated as minimizing the vessel turnover time through well-coordinated operations, and in turn maximizing the throughput, i.e., handled containers. For the quay crane assignment and scheduling problem, the precedence relations of the containers, the safety distances between quay cranes and the traveling times of the quay cranes are considered. For the yard location assignment and container stacking problems, the workloads of the yard cranes, the distances between the quay side and the storage vard locations, and several container stacking policies are respected. The vehicle dispatching problem is employed to handle the transportation of containers between the yard and quay sides. For this problem, both the unloaded containers from the vessels, and the loaded ones onto the vessels are planned simultaneously to reduce the transportation times inside the terminal. The integrated problem is formulated as a constraint programming (CP) model, since rich modeling tools are provided by CP to represent complex combinatorial problems. Through the implementation of CP, which uses constraints to infer new constraints, the domains of problem variables can be reduced by eliminating certain values that violate the inferred constraints. Although there has been some previous effort in constraint programming models for some problems regarding terminal operations, the existing studies only integrate crane handling and truck transportation. In our study, we aim to propose an aggregate plan for all interrelated operations that need to be solved efficiently, in order to reflect the complete terminal system. The main contribution of our study therefore lies in the formulation of a novel CP model that integrates the quay and yard operations at container terminals. As this holistic approach brings an additional complexity due to integration of several connected problems, a heuristic approach is also developed for obtaining fast approximate solutions to the integrated problem. Robust and applicable plans are aimed to be obtained through the developed heuristic. Numerical experiments are conducted to test the performance of the proposed model and solution approach. Effective cohesive plans can be generated in short computational times for the studied problems, which is especially critical for the dynamic environment of container terminals.

# Robust storage loading problems with stacking constraints

Thanh Le Xuan

Department of Mathematics and Computer Science, Osnabrueck University, xuanthanh.le@uni-osnabrueck.de

#### Sigrid Knust

Department of Mathematics and Computer Science, Osnabrueck University, sigrid@informatik.uni-osnabrueck.de

We consider storage loading problems under uncertainty where the storage area is organized in fixed stacks with a limited height. Such problems appear in several practical applications, e.g., when loading container terminals, container ships or warehouses [3]. Incoming items arriving at a partly filled storage area have to be assigned to stacks regarding that not every item may be stacked on top of every other item and taking into account uncertain data of items arriving later. Following the robust optimization paradigm [1,2], we study complexity results in some particular cases and propose different MIP formulations for the strictly and adjustable robust counterparts of the uncertain problem. Furthermore, we show that in the case of interval uncertainties the computational effort to find adjustable robust solutions can be reduced. Computational results on randomly generated instances are presented showing that including robustness improves solutions which do not take uncertainty into account.

#### References

- 1 A. Ben-Tal, L. El Ghaoui, and A. Nemirovski (2009). Robust Optimization. Princeton University Press, Princeton and Oxford.
- 2 A. Ben-Tal, A. Goryashko, E. Guslitzer, and A. Nemirovski(2003). Adjustable robust solutions of uncertain linear programs. Math. Programming A 99(2):351-376.
- 3 J. Lehnfeld and S. Knust (2014). Loading, unloading and premarshalling

of stacks in storage areas: Survey and classication. European Journal of Operational Research 239(2):297-312.

### Loading containers with boxes: the ESICUP Renault challenge

**Olivier Briant** 

Laboratoire G-SCOP Grenoble INP, olivier.briant@grenoble-inp.fr

**Denis Naddef** 

Laboratoire G-SCOP Grenoble INP, denis.naddef@gmail.com

Last February ended the ESICUP-Renault Challenge which consisted in writing a software to load containers with car components, fit into boxes, for expedition to the different Renault plants around the world. After describing the problem object of this challenge, we will briefly describe in this talk the algorithm that has led us to win the challenge.

### An iterated local search algorithm for the bin packing problem with generalized precedence constraints

Raphael Kramer UNIMORE, raphael.kramer@unimore.it

Mauro Dell'Amico UNIMORE, mauro.dellamico@unimore.it

Manuel Iori UNIMORE, manuel.iori@unimore.it

In this paper we propose a generalization of the bin packing problem, obtained by adding precedences between items that can assume heterogeneous non-negative integer values. Such generalization also models the well-known Simple Assembly Line Balancing Problem of type I. To solve the problem we propose a simple and effective iterated local search algorithm that integrates two constructive procedures and four neighborhood structures to guide the search to local optimal solutions. Moreover, we apply some preprocessing procedures and adapt classical lower bounds from the literature. Computational experiments on benchmark instances suggest that the developed algorithm is able to generate good solutions in a reasonable computational time.

### On vertices and facets of combinatorial 2-level polytopes

Manuel Aprile EPFL, manuel.aprile@epfl.ch

Alfonso Cevallos EPFL, alfonso.cevallosmanzano@epfl.ch

> Yuri Faenza EPFL, yuri.faenza@epfl.ch

2-level polytopes naturally appear in several areas of mathematics, including combinatorial optimization, polyhedral combinatorics, communication complexity, and statistics.

We investigate upper bounds on the product of the number of facets  $f_{d-1}(P)$  and the number of vertices  $f_0(P)$ , where d is the dimension of a 2level polytope P. This question was first posed in Bohn et al. (2015), where experimental results showed  $f_0(P)f_{d-1}(P) \leq d2^{d+1}$  up to d = 6.

We show that this bound holds for all known (to the best of our knowledge) 2-level polytopes coming from combinatorial settings, including stable set polytopes of perfect graphs and all 2-level base polytopes of matroids. For the latter family, we also give a simple description of the facet-defining inequalities. These results are achieved by an investigation of related combinatorial objects, that could be of independent interest.

#### References

 A. Bohn, Y. Faenza, S. Fiorini, V. Fisikopoulos, M. Macchia, and K. Pashkovich (2015) Enumeration of 2-level polytopes. In Algorithms -ESA 2015, volume 9294 of Lecture Notes in Computer Science, 191-202.

# Two-level polytopes with a prescribed facet

Samuel Fiorini

Université Libre de Bruxelles, sfiorini@ulb.ac.be

Vissarion Fisikopoulos Computer Science Department, Université Libre de Bruxelles, vfisikop@ulb.ac.be

Marco Macchia

Université Libre de Bruxelles, mmacchia@ulb.ac.be

A (convex) polytope is said to be 2-level if for every facet-defining direction of hyperplanes, its vertices can be covered with two hyperplanes of that direction. These polytopes are motivated by questions, e.g., in combinatorial optimization and communication complexity. We study 2-level polytopes with one prescribed facet. Based on new general findings about the structure of 2-level polytopes, we give a complete characterization of the 2-level polytopes with some facet isomorphic to a sequentially Hanner polytope, and improve the enumeration algorithm of Bohn et al. (ESA 2015). We obtain, for the first time, the complete list of d-dimensional 2-level polytopes up to affine equivalence for dimension d=7. As it turns out, geometric constructions that we call suspensions play a prominent role in both our theoretical and experimental results. This yields to exciting new research questions on 2-level polytopes, which we state in the paper.

#### References

[1] A. Bohn, Y. Faenza, S. Fiorini, V. Fisikopoulos, M. Macchia, and K. Pashkovich. Enumeration of 2-level polytopes. In Nikhil Bansal and Irene Finocchi, editors, Algorithms - ESA 2015, volume 9294 of Lecture Notes in Computer Science, pages 191-202. Springer Berlin Heidelberg, 2015.
### Toward computer-assisted discovery and automated proofs of cutting plane theorems

Matthias Koeppe

University of California, Davis, Dept. of Mathematics, mkoeppe@math.ucdavis.edu

#### Yuan Zhou

University of California, Davis, Dept. of Mathematics, yzh@math.ucdavis.edu

Using a metaprogramming technique and semialgebraic computations, we provide computer-based proofs for old and new cutting-plane theorems in Gomory–Johnson's model of cut generating functions.

# Survivable networks with high connectivity requirements: valid inequalities and branch-and-cut

Meriem Mahjoub

Université Paris Dauphine, meriem.mahjoub@lamsade.dauphine.fr

#### Ibrahima Diarrassouba

Laboratoire de Mathématiques Appliquées du Havre - Université du Havre, diarrasi@univ-lehavre.fr

#### Ridha Mahjoub

LAMSADE, University Paris-Dauphine, mahjoub@lamsade.dauphine.fr

We consider the k-node connected subgraph problem. We propose an integer linear programming formulation for the problem and investigate the associated polytope. We introduce further classes of valid inequalities and discuss their facial aspect. We also devise separation routines and discuss some reduction operations that can be used in a preprocessing phase for the separation. Using those results, we devise a Branch-and-Cut algorithm and present some preliminary computational results.

# A novel MILP formulation and bounds for the makespan minimization problem on assembly lines

Sel Ozcan

Yasar University, sel.ozcan@yasar.edu.tr

Deniz Türsel Eliiyi

Yasar University, deniz.eliiyi@yasar.edu.tr

#### Levent Kandiller

Yasar University, levent.kandiller@yasar.edu.tr

Due to significant changes in product requirements, companies have to configure customized production systems for their products. Assembly lines are flow-line systems developed for high-quantity production of customized products. In addition to their importance on mass-customization, assembly lines are critical for low volume production. Among various decision problems in production planning, assembly line balancing problem plays a critical role in both tactical and operational level in most make-to-order systems (Scholl and Becker, 2006). Hence, configuring efficient make-to-order systems, and decreasing the makespan of the order quantity Q, i.e.,  $C_{max}(Q)$  for keeping up with strict due dates is crucial to stay competitive. The makespan is defined as the time interval from the start of the first task of the order to the termination of the last task. The Simple Assembly Line Balancing Problem-2 (SALBP-2) deals with an assembly line consisting of K stations connected by a material handling system, on which product units Q are launched at a constant rate, C. To complete all tasks and satisfy the precedence relations among tasks, a product unit must visit all stations, and spend exactly C time units at each station. Thus, the first product unit spends CK time units in the line, whereas the rest of the products spend C time units each; one product unit is produced in every cycle. Then, the makespan of any order quantity Q can be computed as  $C_{max}(Q) = C(K) + C(Q-1)$ . SALBP-2 reduces to the identical parallel machine scheduling problem with makespan minimization  $(P_m \| C_{max})$  when precedence relations are ignored, providing a lower bound for C. To ensure precedence constraints, a complex part flow among stations, where a part needs to visit a station more than once, might be required. For the case of small-sized and light-weight products, ignoring precedence constraints by allowing irregular part flows might reduce C, and improve  $C_{max}(Q)$  in the expense of revisiting the stations. We propose a novel MILP formulation minimizing  $C_{max}(Q)$ , which dominates both of the classical SALBP-2 and  $P_m \| C_{max}$  formulations under a tandem cyclic layout setting, where flow is allowed from the last station to the first one. C is at its maximal value when every station is visited once. Conversely, a lower bound for the C is obtained by allowing station re-visits, reducing to the problem  $P_m || C_{max}$ . However, there is a certain range of Q for which the proposed model dominates the other two. The results of the computational study show that SALBP-2 outperforms for small order quantities, whereas  $P_m \| C_{max}$  yields the best results for larger order quantities. For a considerable range of Q, the proposed model yields the best  $C_{max}(Q)$  values. In addition to the novel formulation, bounds and an iterative approach are developed for the defined problem. Results are observed over several numerical examples.

# Robust optimization for ressource-constrained project scheduling in the French Procurement Agency (DGA)

Lucas Hassan ENSTA ParisTech

Nicolas Dupin University of Lille, nicolas.dupin@intradef.gouv.fr

> **Rémi Parize** DGA/CATOD

The main activity of the French Procurement Agency (DGA) is to prepare the future of Defence systems and to procure equipments for the Armed Forces. Operations Research (OR) and System Engineering (SE) are complementary to support decision-makers to rationalize decisions. We present a conjoint work of the OR service CATOD with the management program SC-COA, to optimize scheduling decisions to dismantle old air defence systems. A first extension of the academic resource-constrained project scheduling problem RCPSP incorporates costs considerations. In an uncertain context, where several milestones can be delayed, robust optimization is an appropriate framework for the decision makers problematics. More precisely, the approach of Bertsimas and Sim is useful to handle cost uncertainty, and the Light Robustness approach of Fischetti et al allows to handle more general uncertainties. These approaches tackle the real-world instances, providing an innovative management tool stack in the DGA.

# Unrelated parallel machine scheduling problem with precedence constraints: polyhedral analysis and branch-and-cut

Mohammed-Albarra Hassan Universite de Lorraine, barra\_hassan@hotmail.com

Imed Kacem Universite de Lorraine, imed.kacem@univ-lorraine.fr

Sebastien Martin Universite de Lorraine, sebastien.martin@univ-lorraine.fr

### Izzeldin M.Osman

Sudan University of Science and Technoloy, izzeldin@acm.org

We consider the problem of unrelated parallel machines with precedence constraints (UPMPC), with the aim of minimizing the makespan. Each task has to be assigned to a unique machine and no preemption is allowed. In this paper, we focus on the problem of interval and *m*-clique free subgraphs. We show the relation between the graph problem and the UPMSPC problem. We propose valid inequalities and study the facial structure of their polytope. Facets are presented to bound the associated integer linear program formulation to help in solving the global problem. We develop a Branch and Cut algorithm for solving the problem and present some experimental results.

## A branch-and-check approach to solve a wind turbine maintenance scheduling problem

Aurélien Froger UCO-IMA, froger.aurelien@gmail.com

Michel Gendreau CIRRELT and MAGI, École Polytechnique de Montréal, Michel.gendreau@cirrelt.net

Jorge E. Mendoza Université François-Rabelais de Tours, jorge.mendoza@univ-tours.fr

Eric Pinson

IMA/UCO, Eric.PINSON@uco.fr

Louis-Martin Rousseau

École Polytechnique de Montréal, louis-martin.rousseau@polymtl.ca

Driven by climate change mitigation efforts, wind energy has significantly increased in recent years. In this context, efficient wind turbine maintenance planning and scheduling becomes a critical tool to prevent unnecessary downtime and excessive operational costs. We discuss here a challenging maintenance scheduling problem rising in wind farms. We address the problem on a short-term horizon considering an individual management of the technicians through a space-time tracking. The objective is to find a maintenance plan that maximizes the production of the turbines while taking into account wind predictions, multiple task execution modes, and task-technician assignment constraints.

We solve the problem taking advantage of its intrinsic decomposition into a scheduling problem on the one hand and into a resource management problem on the other hand. The scheduling problem consists in building a maintenance planning in order to maximize the wind energy production. If we assume a fixed maintenance planning, the resource management problem checks if the technician requests can be satisfied while meeting the location-based incompatibility constraints and coping with individual resource availability

periods. Note that an optimal solution of the scheduling problem leading to a feasible resource management problem is optimal for the whole problem. To efficiently solve the problem while exploiting this decomposition, we develop a branch-and-check approach. Since the sub-problem does not possess the integrality property, we invalidate infeasible solutions of the relaxed master problem using combinatorial Benders' cuts. However, generating cuts stating that at least one variable of the master problem must change value may lead to a very slow convergence, so we use those cuts as a last resort. We solve beforehand the linear relaxation of the sub-problem to identify potential violated Benders feasibility cuts. We also build up alternative cuts by approximating the sub-problem by a set of b-matching problems as well as by a set of maximum-weight clique problems. These approximations allow us to potentially identify multiple cuts. Since these cuts are expressed using a reduced number of variables of the master problem, they produce stronger combinatorial Benders' cuts too. To speed-up the process, we also generate cuts for non-integer nodes. Last but not least, since the master problem is likely to be difficult to solve, we first solve its linear relaxation using a classic Benders decomposition approach.

We report results on randomly generated instances built with input from wind forecasting and maintenance scheduling software companies. Within a time limit of 3 hours, the method shows an average gap of 1% with respect to the optimal solutions if known (half of the instances are optimally solved), or to the best found upper bounds otherwise. It significantly outperforms the direct resolution of an integer linear programming model.

## A matheuristic algorithm for the multi-depot inventory routing problem

Demetrio Laganà

University of Calabria, demetrio.lagana@unical.it

#### Annarita De Maio

University of Calabria, annaritademaio@gmail.com

In this paper a matheuristic for the solution of an Inventory Routing Problem in a Mega-city context is presented. In order to face the difficulties related to the real context, the urban space is partitioned into districts allowing to create clusters and to generate sets of feasible routes for the IRP.In the scientific literature similar approaches were proposed by. Micheal et al.(2012),and by Campbell et al.(2004).With respect to the state of the art, we consider a different context (Mega-cities).Precisely,more emphasis is devoted to balance simultaneously several factors that impact on clustering and route construction phases: distance customer-depot,demand and inventory customers levels,time horizon extension,vehicle capacity.

Algorithm description

To formally describe the algorithm let  $I = \{1, \ldots, I\}$  be the set of depots in the urban area and let  $J = \{1, \ldots, J\}$  be the set of customers. The problem is defined over an undirected graph  $G = (I \cup J, \mathcal{E})$  (the transportation network). A transportation cost  $c_{ij}$  is associated with each edge  $(i, j) \in \mathcal{E}$ . The aggregation of the customers into clusters is guided by the following factors:maximum cardinality of a cluster, denoted by CC; an average measure related to the number of the critical customers in a cluster, denoted by TC; the critical level of each customer  $v_i$  (it is computed by considering either the number of deliveries in the time horizon to avoid stock-out, or the average delivery cost to service the customer). The proposed algorithm is described in the following: Step 1. Solve an integer programming problem to create clusters. The binary decision variable  $x_{ij}$  is equal to one if the customer j is associated to the depot i.

$$\min \sum_{(i,j)\in\mathcal{E}} (c_{i,j} x_{i,j}) \tag{1}$$

s. to

$$\sum_{j \in \mathcal{J}} (x_{i,j}) \le CC \qquad \qquad \forall i \in \mathcal{I} \tag{2}$$

$$\sum_{i \in \mathcal{I}} (x_{i,j}) = 1 \qquad \qquad \forall j \in \mathcal{J} \tag{3}$$

$$\sum_{j \in \mathcal{J}} (v_j / CC) x_{i,j} \le TC \qquad \forall i \in \mathcal{I}$$
(4)

$$x_{i,j} \in (0,1) \tag{5}$$

Objective function allows to associate each customer to the nearest depot.Constraints (2) impose a maximum cluster cardinality;constraints (3) associate each customer to a single depot;constraints (4) impose that each cluster must be composed by critical and not critical customers. Step 2. A feasible routes construction phase for each cluster is performed through different criteria:intra-cluster route construction;inter-clusters route construction;intraclusters large route construction designed on the base of the vehicle capacity,the inventory levels,the demands of each customer.

### References

- S. Michel, F. Vanderbeck, A Column-Generation Based Tactical Planning Method for Inventory Routing, Operations Research Vol.60 No.2, pp.382–397,2012
- [2] A.M Campbell, M.W.P. Savelsbergh, A Decomposition Approach for the Inventory-Routing Problem, Transportation Science, Vol.38 No.4, pp.488–502, 2004

### Modelling and solving the joint order batching and picker routing problem in inventories

Cristiano Arbex Valle

UFMG - Universidade Federal de Minas Gerais, arbex@dcc.ufmg.br

John E. Beasley

Brunel University, john.beasley@brunel.ac.uk

#### Alexandre Salles Da Cunha

UFMG - Universidade Federal de Minas Gerais, acunha@dcc.ufmg.br

In this work we investigate the problem of order batching and picker routing in inventories. These are labour and capital intensive problems, often responsible for a substantial share of warehouse operating costs. In particular, we consider the case of online grocery shopping in which orders may be composed of dozens of items. To the best of our knowledge, no exact algorithms have been proposed for this problem. We therefore introduce three integer programming formulations for the joint problem of batching and routing, one of them involving exponentially many constraints to enforce connectivity requirements and two compact formulations based on network flows. For the former we implement a branch-and-cut algorithm which separates connectivity constraints. We built a test instance generator, partially based on publicly-available real world data, in order to compare empirically the three formulations.

### Vehicle routing problem with drones: worst-case bounds and related problems

Stefan Poikonen

University of Maryland - College Park, spoikone@math.umd.edu

Xingyin Wang

University of Maryland - College Park, wangxy@math.umd.edu

Bruce Golden

University of Maryland - College Park, bgolden@rhsmith.umd.edu

The vehicle routing problem with drones (VRPD) is inspired by the increasing interest in commercial drone delivery by companies such as Amazon, Google, DHL, and WalMart. In this model, a fleet of m homogenous trucks each carries k drones with a speed of  $\alpha$  times that of the truck. Each drone may dispatch from the top of the truck and carry a package to a customer location. The drone then returns to the top of its truck to recharge or swap batteries (we assume instantaneously). The truck itself is allowed to move and deliver packages, but must be stationary at a delivery location or the depot when launching or retrieving drones. The goal is to minimize the completion time to deliver all packages and return all vehicles back to the central depot.

We establish a number of tight worst-case bounds, displaying the maximum theoretical speed-up with this model compared to traditional VRP or TSP models. Initially we assume trucks and drones are both bound to the distance metric defined by the street network. Bounds are given which vary according to the number of drones per truck and the relative speed of drones to trucks. One such bound is that at maximum VRPD can achieve a  $(\alpha k + 1)$ -fold speed up compared to traditional min-max VRP. We then analyze the cost associated with this model, and explicitly consider the problem of limited drone battery and range. We then consider the impact of utilizing various distance metrics including as-the-crow-flies for drones and generalize some bounds to any arbitrary distance metric, such as:

$$\frac{Z(\text{TSP}, Q_t)}{Z(\text{VRPD}_{m,\alpha}, Q_d)} \le \frac{Z(\text{TSP}, Q_t)}{Z(\text{TSP}, Q_d)} m(\alpha k + 1)$$

where  $Q_t$  and  $Q_d$  are the truck and drone distance metrics respectively, and Z(P) is the optimal objective value of problem P.

Next we relate the VRPD with the close-enough vehicle routing problem (CEVRP). Not only do we show the problems are equivalent in the limit:

$$lim_{\alpha \to \infty} Z(VRPD_{m,\alpha}) = Z(CEVRP_m)$$

but we also suggest that a solution method to a generalized VRP or to CEVRP can be adapted to give a practical solution heuristic to the VRPD.

The VRPD model has some practical advantages. The trucks effectively extend the range of drones. The drones allow the truck to "parallelize" tasks and are able to take advantage of as-the-crow-flies distance.

### A matheuristic for the multi-vehicle inventory routing problem

Claudia Archetti

Department of Economics and Management, University of Brescia, claudia.archetti@unibs.it

#### Natashia Boland

School of Industrial and Systems Engineering, Georgia Institute of Technology, natashia.boland@isye.gatech.edu

#### M. Grazia Speranza

Department of Economics and Management, University of Brescia, grazia.speranza@unibs.it

In this paper, we consider the MIRP variant in which a supplier must replenish a set of customers, over a discrete time horizon, composed of a given number of days, with a limited fleet of capacitated vehicles. The demand of each customer, and the supply available, in each day, are given. Each customer has an inventory capacity and no stock-out is allowed at the customers. Stock-out is also not permitted at the supplier. When a customer is replenished in advance of its demand, the product received is stocked in inventory. The objective is to minimize the sum, over the time horizon, of the inventory cost at the supplier and at the customers, and the routing cost. We present a matheuristic where different mathematical programming models (MILPs) are embedded in a heuristic scheme. In particular, MILPs are used in two phases of the algorithm. In the initial phase, a MILP is solved to obtain a feasible solution to the problem. The MILP is obtained by relaxing the original problem formulation. Then, a tabu search phase follows. A second MILP is solved at the final phase of the algorithm to improve the solution provided by the tabu search. Computational results are presented for a large set of benchmark instances and compared with state of the art results. When compared with two exact methods on 640 small instances, the matheuristic finds 192 (48%) optima over the 402 instances with known optima and improves 125 upper bounds. Tested on 240 large instances, (with up to 200 customers), for which no optimal solutions are known, it improves the best solution for 220 (92)

### OR and AI methods to solve diameter and degree constrained network design problems

Deepak Mehta

Insight Centre for Data Analytics, University College Cork, deepak.mehta@insight-centre.org

Barry O'Sullivan Insight Centre for Data Analytics, University College Cork, b.osullivan@cs.ucc.ie

#### Cemalettin Ozturk

Insight Centre for Data Analytics, University College Cor, cemalettin.ozturk@insight-centre.org

#### Luis Quesada

Insight Centre for Data Analytics, University College Cork, l.quesada@4c.ucc.ie

The next generation broadband networks must provide a very high speed broadband capability to all users, and it should remain economically viable, energy efficient, and environmentally sustainable. In order to realise this goal a Transparent Optical Core Network (TOCN) is specified as one of the key requirements of such a next generation network developed by the DISCUS project (an EU-funded project), which can significantly reduce the energy bills of operating such a core network. Among the requirements of a TOCN we have that the diameter and the degree of the network should be constrained by given thresholds. The objective in the design of a TOCN is to minimise the size of the network. Modelling and solving this combinatorial optimisation problem using state of the art solvers does not scale in terms of memory and time even for smaller size network instances. We therefore propose two novel algorithms based on Operations Research (OR) and Artificial Intelligence (AI) methods. The OR method exploits the structural properties of the mixed integer programming model of the problem. The AI method uses the QuickXplain algorithm to find a solution through the computation of a minimal set of links. Our empirical evaluations, based on real countrywide

networks, show that our approaches scale well without hampering the quality of the solution.

### Benders decomposition for capacitated network design

Sara Mattia IASI-CNR, sara.mattia@iasi.cnr.it

We consider the problem of choosing the edges to be activated in a capacitated network to guarantee the routing of a set of commodities. Two routing policies are investigated: splittable (unrestricted routing) and unsplittable (single path routing). We study, both from the theoretical and from the computational point of view, a Benders-like formulation of the problem. We present polyhedral results showing that, contrary to what happens for the capacity allocation version of the problem, the Benders-like formulation is not completely defined by inequalities having metric left-hand-side coefficients. We focus on inequalities generated by subproblems obtained by partitioning the node set. We gave a condition stating when a facet of the p-node problem can be extended to a facet of the original problem. We also prove that for splittable flows the well-known cutset inequalities remain facets for the capacitated problem, whereas this is not true for unsplittable flows. As for the computational investigation, a preliminary testing shows that on average the Benders formulation is feaster than the other two approaches, especially when the capacity increases. Moreover, for unsplittable flows it is less affected by the increasing of the control parameters (edge, nodes and commodities) than the flow formulation.

## On a general framework for network representability in discrete optimization

#### Yuni Iwamasa

The University of Tokyo, yuni\_iwamasa@mist.i.u-tokyo.ac.jp

In discrete optimization, representing an objective function as an *s*-*t* cut function of a network is a basic technique to design an efficient minimization algorithm. A network representable function can be minimized by computing a minimum *s*-*t* cut of a directed network, which is a very easy and fastly solved problem. Hence it is natural to ask what functions are network representable. In the case of pseudo Boolean functions (functions on  $\{0,1\}^n$ ), it is known that any submodular function of arity at most 3 is network representable. Živný-Cohen-Jeavons showed by using the theory of the valued constraint satisfaction problem (VCSP) that a certain submodular function of arity 4 is not network representable.

In this paper, we aim at developing a network representation theory beyond  $\{0,1\}^n$ . We introduce a general framework for the network representability of functions on  $D^n$ , where D is an arbitrary finite set. We completely characterize network representable functions on  $\{0,1\}^n$  in our new definition. We can apply the VCSP theory to the network representability in the proposed definition. We prove that some ternary bisubmodular function and some binary k-submodular function are not network representable.

### Exact approaches for network design problems with relays

Ivana Ljubic

ESSEC Business School of Paris, ivana.ljubic@essec.edu

Markus Leitner

University of Vienna, markus.leitner@univie.ac.at

Martin Riedler

Vienna University of Technology, riedler@ac.tuwien.ac.at

#### Mario Ruthmair

University of Vienna, mario.ruthmair@univie.ac.at

We consider the Network Design Problem with Relays (NDPR) which gives answers to some important strategic design questions in telecommunication network design and electric mobility. Given a family of origin-destination pairs and a set of existing links in the telecommunication context these questions are: (1) What are the optimal locations for signal regeneration devices (relays) and how many of them are needed? (2) Could the available infrastructure be enhanced by installing additional links in order to reduce the travel distance and therefore reduce the number of necessary relays? In e-mobility relays correspond to charging stations for electric vehicles while additional links reflect roads for which tolls need to be paid.

In contrast to previous work on the NDPR which mainly focused on heuristic approaches, we discuss exact methods based on different mixed integer linear programming formulations for the problem. We develop Branchand-Price and Branch-Price-and-Cut algorithms that build upon models with an exponential number of variables (and constraints). In an extensive computational study, we analyze the performance of these approaches for instances that reflect different real-world settings.

# Min-up/min-down unit commitment problem: complexity and valid inequalities

Pascale Bendotti EDF R&D, pascale.bendotti@edf.fr

 $\begin{array}{c} {\bf C\acute{e}cile\ Rottner}\\ {\it EDF\ R}{\it \earrow}D,\ {\tt cecile.rottner@lip6.fr} \end{array}$ 

Pierre Fouilhoux Laboratoire LIP6, Université Pierre et Marie Curie, pierre.fouilhoux@lip6.fr

The *min-up/min-down Unit Commitment Problem* (UCP) is to find a minimum-cost production schedule on a discrete time interval for a set of thermal units for electricity production. At each time period, the total production should meet a forecasted demand. Each unit must satisfy minimum up-time and down-time constraints besides featuring production and start-up costs.

The UCP has been proven to be NP-hard as a generalization of the knapsack problem. In this article, we show that the UCP is still NP-hard even if every unit produces the same quantity of power.

We consider a MIP formulation based on the inequality introduced by Rajan and Takriti [1]. For the particular case when the UCP is limited to a single unit, it has been proven in [1] that the associated polytope is integral.

We study some polyhedral aspects of the UCP polytope for several units. We first extend the classical *Cover inequalities* of the knapsack polytope to obtain valid inequalities for the UCP polytope: for each period and for each subset C of units, there must be enough units up in C.

We introduce a new class of inequalities, named Interval-Cover inequalities. For a given time interval  $\mathcal{I}$ , such inequalities ensure that for any subset C of units, if  $i \in C$  is down at a given period of  $\mathcal{I}$ , other units in C must be up to compensate for unit i being down. We provide a characterization of the cases in which Interval-Cover inequalities are valid and not dominated by other inequalities. We prove that the separation problem for the Interval-Cover inequalities is NP-complete. We elaborate both exact and heuristic methods to separate Interval-Cover inequalities and we devise a Branch-&-Cut algorithm. Preliminary results are promising on some categories of difficult instances.

### References

1 Deepak Rajan, Samer Takriti. Minimum Up/Down Polytopes of the Unit Commitment Problem with Start-Up Costs. IBM Research Report, 2005

# Ring spur assignment problem: new formulation, valid inequalities and a branch-and-cut approach

Rahimeh Neamatian Monemi

French institute of science and technology for transport, spatial planning, development and networks, r.n.monemi@gmail.com

> **Bernard Fortz** Université Libre de Bruxelles

Shahin Gelareh Univ. d'Artois, shahin.gelareh@gmail.com

We propose a new mathematical model for the Ring Spur Assignment Problem and identify several classes of valid inequalities. In addition we propose separation algorithms for separating those valid inequalities and an efficient branch-and-cut algorithm. Our numerical experiment shows superiority of this model to the previously proposed one in term of computational behavior.

## The multi-terminal vertex separator problem: polytope characterization and TDI-ness

Youcef Magnouche

LAMSADE, Université Paris Dauphine, magnouche.youcef@gmail.com

#### Sébastien Martin

LCOMS, Université de Lorraine, sebastien.martin@univ-lorraine.fr

Let  $G = (V \cup T, E)$  be a simple graph with  $V \cup T$  the set of vertices, where T is a set of distinguished vertices called terminals, inducing a stable set and E a set of edges. Given a weight function  $w: V \to \mathbb{N}$ , the multi-terminal vertex separator problem (MTVSP) consists in finding a subset  $S \subseteq V$  of minimum weight such that each path between two terminals intersects S. The problem can be solved in polynomial time when |T| = 2, [1] but when |T| > 3, the MTVSP is NP-complete, [2]. In this paper we deal with the MTVSP in two specific classes of graph, star trees and clique stars, showing that this problem can be solved in polynomial time for any size of T in these two classes. We show also that the associated polytope is integer and we give a minmax relation for each class. The MTVS problem has applications in different areas like VLSI conception, linear algebra, connectivity problems and parallel algorithms. It has also applications in network security. The MTVS problem was considered in [2], in which the authors present several valid inequalities and develop a branch-and-cut algorithm to solve the problem. They also present two classes of graph, called star trees and clique stars, on which our work is based. In [3], authors give a linear system for the MTVSP and characterize the class of graph for which it is total dual integral for any size of T, i.e., the dual problem has an integer optimal solution for any integer vertex weight vector. We would like point out that these class of graph does not contain some star trees and clique stars. In this paper we first characterize the polytope of the multi-terminal vertex separators in these two classes of graph and then we give TDI linear systems.

### References

- [1] Ben-Ameur, W., Didi Biha, M.: On the minimum cut separator problem, Networks, 59 (1), pages 30-36 (2012).
- [2] Cornaz, D., Magnouche, Y., Mahjoub, A.R., Martin, S.: The multiterminal vertex separator problem : Polyhedral analysis and branch-andcut., International Conference on Computers and Industrial Engineering (2015).
- [3] Jost, V., Naves, G.: The graphs with the max-Mader-flow-min-multiwaycut property, Journal Computing Research Repository - CORR, vol. abs/ 1101.2 (2011).
- [4] Schrijver, A.: Combinatorial Optimization: Polyhedra and Efficiency, (2003).

## A full description of polytopes related to the index of the lowest nonzero row of an assignment matrix

Walid Ben-Ameur

*Telecom SudParis*, walid.benameur@telecom-sudparis.eu

Antoine Glorieux Telecom SudParis, glorieux@telecom-sudparis.eu

Jose Neto Telecom SudParis, jose.neto@telecom-sudparis.eu

Consider a {0,1} assignment matrix where each column contains exactly one coefficient equal to 1 and let h be the index of the lowest row that is not identically equal to the zero row. We give a full description of the convex hull of all feasible assignments appended with the extra parameter h. This polytope and some of its variants naturally appear in the context of several combinatorial optimization problems including frequency assignment, job scheduling, graph orientation, maximum clique, etc. We also show that the underlying separation problems are solvable in polynomial time and thus optimization over those polytopes can be done in polynomial time.

# Using exact subgraph constraints for improving the Lovász theta function as bound on the stability number and the coloring number

Elisabeth Gaar

Alpen-Adria-Universitaet Klagenfurt, elisabeth.gaar@aau.at

#### Franz Rendl

Alpen-Adria-Universitaet Klagenfurt, franz.rendl@aau.at

The stable set problem and the coloring problem are both well-known combinatorial optimization problems. They are both NP-complete and hard to approximate. Nevertheless one wants to solve big instances of these problems and therefore tight upper bounds on the stability number  $\alpha(G)$  of a graph G and tight lower bounds on the chromatic number  $\chi(G)$  are needed. One possible bound is given by the Lovász theta function  $\vartheta(G)$ , namely

 $\alpha(G) \leqslant \vartheta(G) \leqslant \chi(\overline{G})$ 

holds. The Lovász theta function can be computed in polynomial time to arbitrary precision as semidefinite program (SDP).

We consider two different formulations of  $\vartheta(G)$  and  $\vartheta(\overline{G})$  as SDP; a maximization problem as an upper bound on  $\alpha(G)$  and a minimization problem as a lower bound on  $\chi(G)$ . In order to improve these bounds, to the SDPs we add additional so called exact subgraph constraints, which were introduced in a technical report by Adams, Anjos, Rendl and Wiegele.

If the squared stable set polytope  $STAB^2(G)$  and the coloring polytope COL(G) of a graph G are defined as

$$\begin{aligned} \operatorname{STAB}^2(G) &:= \operatorname{conv}\{ss^T \mid \text{s incidence vector of a stable set of } G\} \text{ and} \\ \operatorname{COL}(G) &:= \operatorname{conv}\{SS^T \mid S \text{ stable set partition matrix of } G\}, \end{aligned}$$

then the exact subgraph constraint for a subgraph H of G are defined as  $X_H \in \text{STAB}^2(H)$  and  $X_H \in \text{COL}(G)$  respectively, where  $X_H$  is the submatrix

of X which corresponds to the vertices of H and the matrix X is a matrix variable of the SDP of  $\vartheta(G)$  and  $\vartheta(\overline{G})$  respectively.

The routine to improve the bounds first calculates  $\vartheta(G)$  and  $\vartheta(\overline{G})$  respectively. Then iteratively some subgraphs are chosen and the corresponding exact subgraph constraints are added as constraints to the SDP.

Obviously one of the most important questions is how to find violated subgraphs, that is subgraphs which are not in the corresponding polytope. One possibility to find violated subgraphs is to try to find a submatrix  $X_H$ of X, such that the inner product of  $X_H$  and a certain matrix A is small. Possible good choices of A are extreme copositive matrices and facet inducing matrices.

In a first implementation for the stable set problem we get good results on many different graph classes, for example random graphs from the Erdős– Rényi-Model and instances of the 2. DIMACS challenge of moderate size. However, unless P = NP, we are not able to expect good results for all graphs. In fact, it turns out that the approach does not work very good for Paley graphs, because in their case there are no violated subgraphs of small size.

### Lovász-Schrijver PSD-operator on claw-free graphs

Silvia Bianchi

Facultad de Ciencias Exactas, Ingeniería y Agrimensura. UNR, sbianchi@fceia.unr.edu.ar

Mariana Escalante

Universidad Nacional de Rosario-CONICET, mariana@fceia.unr.edu.ar

Graciela Nasini

Universidad Nacional de Rosario-CONICET, nasini@fceia.unr.edu.ar

#### Annegret Wagler

University Clermont-Ferrand II (Blaise Pascal), wagler@isima.fr

The context of this work is the study of the stable set polytope, some of its linear and semi-definite relaxations, and graph classes for which certain relaxations are tight. Our focus lies on the positive semi-definite operator  $LS_+$  introduced by Lovász and Schrijver [4]. The weighted stable set problem can be solved in polynomial time for graphs where a single application of  $LS_+$ to the edge relaxation yields the stable set polytope. These graphs are called  $LS_+$ -perfect (or  $N_+$ -perfect, as appeared in [2,3]).

In [1], a conjecture is formulated about a polyhedral characterization of  $LS_+$ -perfect graphs claiming that all facet defining inequalities of their stable set polytope have near-bipartite support. This conjecture has been already verified for some graph classes, including webs [2] and line graphs [3]. Our contribution is to extend its validity to a well-studied class containing all webs and line graphs: the class of claw-free graphs.

The results have parallels to the well-developed research area of perfect graph theory, combining and using a relatively wide group of existing results related to the stable set polytope and its various convex relaxations.

### References

[1] S. Bianchi, M. Escalante, G. Nasini, and L. Tunçel, "Lovász-Schrijver SDP-operator, near-perfect graphs and near-bipartite graphs". Submitted manuscript. Available in arXiv:1411.2069v1 [cs.DM] (2014).

- [2] M. Escalante and G. Nasini, "Lovász and Schrijver  $N_+$ -relaxation on web graphs," *Lecture Notes in Computer Sciences*, vol. 8596 (ISCO 2014), pp. 221–229, 2014.
- [3] M. Escalante, G. Nasini, and A. Wagler, "Characterizing  $N_+$ -perfect line graphs", to appear in International Transactions in Operational Research, 2016.
- [4] L. Lovász and A. Schrijver, "Cones of matrices and set-functions and 0-1 optimization," SIAM J. on Optimization, vol. 1, pp. 166–190, 1991.

### Strengthening Chvatal-Gomory cuts for the stable set problem

Adam N. Letchford

Department of Management Science - Lancaster University, a.n.letchford@lancaster.ac.uk

Francesca Marzi

Department of Engineering and Information Sciences and Mathematics -University of L'Aquila, francesca.marzi@graduate.univaq.it

#### Fabrizio Rossi

Department of Engineering and Information Sciences and Mathematics -University of L'Aquila, fabrizio.rossi@univaq.it

#### Stefano Smriglio

Department of Engineering and Information Sciences and Mathematics -University of L'Aquila, stefano.smriglio@univaq.it

The stable set problem is a well-known NP-hard combinatorial optimization problem. As well as being hard to solve (or even approximate) in theory, it is often hard to solve in practice. The main difficulty is that upper bounds based on linear programming (LP) tend to be weak, whereas upper bounds based on semidefinite programming (SDP) take a long time to compute. We propose a new method to strengthen the LP-based upper bounds. The key idea is to take violated Chvatal-Gomory cuts and then strengthen their right-hand sides. Although the strengthening problem is itself NP-hard, it can be solved reasonably quickly in practice. As a result, the overall procedure proves to be capable of yielding competitive upper bounds in reasonable computing times.

# Hybrid genetic algorithm with local search for the multi-vehicle covering tour problem

Manel Kamoun MODILS, kamoun\_manel@yahoo.com

Houda Derbel MODILS, derbelhouda@yahoo.fr

Bassem Jarboui MODILS, bassem\_jarboui@yahoo.fr

This paper deals with the multi-vehicle covering tour problem wich is a generalization of the vehicle routing problem where the constraint of visiting all custumers is not valid. We consider a hybrid genetic algorithm to solve the problem. Our hybridization consists of generating feasible solutions using the genetic algorithm and improving the resulting ones by a local search method. We tested and compared our hybrid algorithm on datasets based on TSP Library instances.

## A convex programming approach to drone routing with obstacles and physical constraints

#### Stefan Poikonen

University of Maryland, spoikone@math.umd.edu

In this paper, we wish to route a drone between two given points in the minimum amount of time possible, while obeying a number of physical constraints (finite velocity, acceleration, and curvature, etc.) and avoiding a number obstacles that are initially modeled as circular radar zones (but then generalized to include arbitary shapes).

To begin, we construct a Voronoi diagram around the radar centers. Any Voronoi edge intersecting with a radar zone is given a weight of  $\infty$ . We then apply Dijkstra's Algorithm on the Voronoi graph to give us an initial path. This path, however, contains non-smooth turns which violate curvature constraints. Set T to be some unrestrictively large completion time. We then discretize this Voronoi edge path into K waypoints. These waypoints are separated by a timestep of h = T/K.

We then repeat a core iterative step. The broad notion is that we construct a convex program with objective min(0) then perform a feasibility test. We seek to find a feasible set of K waypoints, each separated by time h (and therefore with a total completion time of T). If a feasible solution is found, then we reduce T and h by the same factor, and again attempt to find a feasible set of waypoints, although with a smaller allowed completion time T. If we cannot find a feasible solution, we choose a more conservative value for T and attempt to find a feasible solution.

Embedded in the convex program are numerous constraints. Firstly, the original physical constraints (regarding velocity, acceleration, etc.) are in the convex program. In addition, we construct a "safety zone" around each way-point. A safety zone is the maximum circular region able to be constructed around a waypoint that does not intersect with any radar zone. Each time we find a feasible solution for a smaller value of T, we recenter our safety zones at these new waypoints. This allows our safety zones to drift, and we are not constrained to our original safety zones.

With slight modification, this method can be extended to avoiding arbitrarily shaped obstacles. In addition, we can extend this method to avoiding dynamic objects with reasonably predictable trajectories. Furthermore, we propose an algorithm for routing multiple drones to avoid static obstacles, dynamic obstacles, and to avoid other drones. Finally we show how we could impose a fleet a formation with a few extra constraints in our convex program.

## The production-distribution problem with order acceptance and package delivery: models and algorithm

Majid Khalili

Karaj Branch, Islamic Azad University, khalili.mj@gmail.com

Hamed Tayebi

Karaj Branch, Islamic Azad University, hamed.tayyebi@gmail.com

#### Mehran Esmailpour

Karaj Branch, Islamic Azad University, mehran.es82@gmail.com

The production planning and distribution are among the most important decisions in the supply chain. Classically, in this problem, it is assumed that all orders have to produced and separately delivered; while, in practice, an order may be rejected if the cost that it brings to the supply chain exceeds its revenue. Moreover, orders can be delivered in a batch to reduce the related costs. This paper considers the production planning and distribution problem with order acceptance and package delivery to maximize the profit. At first, a new mathematical model based on mixed integer linear programming is developed. Using commercial optimization software, the model can optimally solve small or even medium sized instances. For large instances, a solution method, based on imperialist competitive algorithms, is also proposed. Using numerical experiments, the proposed model and algorithm are evaluated.

# Iterative aggregation and disaggregation algorithm for pseudo-polynomial network flow models with side constraints

François Clautiaux

Université des Sciences et Technologies de Lille, francoisclautiaux@gmail.com

#### Said Hanafi

University of Valenciennes, said.hanafi@univ-valenciennes.fr

Rita Macedo Université Lille 3, rita.macedo@univ-lille3.fr

Marie-Emilie VogeINRIA Lille Nord-Europe , marie-emilie.voge@lifl.fr

Cláudio Alves Universidade do Minho, claudio@dps.uminho.pt

In this presentation, we describe a general solution framework based on aggregation techniques to solve NP-Hard problems that can be formulated as a circulation model with specific side constraints. The size of the extended Mixed Integer Linear Programming formulation is generally pseudopolynomial.

To efficiently solve exactly these large scale models, we propose a new iterative aggregation and disaggregation algorithm, which is a generalization of [1] and [2]. The novelty of our proposal is twofold: we describe a more general framework, and an efficient implementation, which is tested on quite two different difficult problems.

An important aspect of our work is its generality. Whereas all aggregation procedures described in previous studies on network-flow problems focus on specific applications, our method applies to a general family of min-cost circulation models which can in turn be applied to a large set of applications. It is based on a given scaling function, which is applied to the arcs' heads and tails. The scaling function determines the level of aggregation as well as the quality of the approximation. It only modifies the possibility of either combining, or not combining, the arcs into feasible paths. As a consequence, a solution that satisfies the side constraints for the original model is obtained directly from the aggregated model. The only question that arises is whether or not the solution respects the flow-conservation constraints. When this is not the case, the method iteratively disaggregates the network until convergence is proved.

The computational experiments on two hard optimization problems (a variant of the vehicle routing problem and the cutting-stock problem) show that a generic implementation of the proposed framework allows us to outperform previous known methods.

### References

- 1 R. Macedo, C. Alves, J. V. de Carvalho, F. Clautiaux, S. Hanafi, Solving exactly the vehicle routing problem with time windows and multiple routes using a pseudo-polynomial model, European Journal of Operational Research 214 (3) (2011) 457–545.
- 2 M.-E. Voge, F. Clautiaux, Theoretical investigation of aggregation in pseudo-polynomial network-flow models, in: A. M. et al. (Ed.), ISCO 2012, 2nd International Symposium on Combinatorial Optimization, Vol. 7422 of LNCS, 2012, pp. 213–224.
## Bilevel model for network interdiction problems

## **Pierre-Louis Poirion**

LIX-Ecole Polytechnique, poirion@lix.polytechnique.fr

Jean-François Baffier

National Institute of Informatics; JST, ERATO, Kawarabayashi Large Graph Project and The Japanese-French Laboratory for Informatics, CNRS, jeanfrancois.baffier@gmail.com

#### Vorapong Suppakitpaisarn

Tokyo University, vorapong@is.s.u-tokyo.ac.jp

We propose to solve the adaptive network flow problem via a bilevel optimization framework. In this problem, we aim to find a flow that is most robust against an optimal k edges attack. There is an exact algorithm proposed to solve the problem in a specific class of input. However, for some input, a flow obtained from the algorithms sometimes much less robust than the optimal one. That motivates us to find an efficient exact algorithm based on bilevel optimization framework for the problem in this paper. Although our framework is still not the most efficient one, this preliminary result can lead us to a better formulation in future.

## Tarder multiflow

Denis Cornaz Paris Dauphine, denis.cornaz@dauphine.fr

> Roland Grappe Paris Nord, grappe@lipn.fr

Mathieu Lacroix Paris Nord, lacroix@lipn.fr

We prove that the cycle cone is box-TDI for series-parallel graphs, this implies a general max-multiflow/min-multicut relation.

# ILP formulations for the railway rescheduling problem under large disruptions

Juan Jose Miranda Bront Universidad de Buenos Aires, jmiranda@dc.uba.ar

Agustin Mosteiro Universidad de Buenos Aires, amosteiro@dc.uba.ar

#### Federico Pousa

Universidad de Buenos Aires, fpousa@dc.uba.ar

Normal railway operations are often influenced by unexpected situations that affect, for example, the rolling stock and the infrastructure and may cause delays and disruptions in the network. Under these scenarios, one of the key issues is to construct a new disposition timetable to recover from the disruption and to inform the passengers as fast as possible, usually within a few minutes. Due to its complexity, the overall problem is usually divided into three phases which are solved sequentially, namely timetable rescheduling, rolling stock rescheduling and crew rescheduling. One of the major issues involves the feasibility of the solution obtained in one phase regarding the constraints imposed by the following one. Therefore, the models try to include, at least in a relaxed fashion, some of the constraints involved in successive phases in order to provide a more robust solution.

There has been a trend in the literature in the last few years to consider MILP as prototypes for automated decision systems to tackle this kind of situations. This research builds mainly upon the works by Louwerse and Huisman [3] and Veelenturf et al. [4]. Louwerse and Huisman [3] consider the timetable rescheduling problem on a railway line, including also some general aspects of the rolling stock rescheduling in the timetabling phase. Veelenturf et al. [4] extend this research by considering networks with a general structure, not necessarily a line, and include as well the possibility of re-routing a train and therefore modifying its original route. In both cases, the results show that the approach produces good quality solutions in reasonable computing times.

In both cases, despite the differences regarding the particular characteristics modeled, the timetable rescheduling problem is formulated by means of an event-activity network which is later translated into an MILP formulation, aiming to obtain the disposition timetable. To the best of our knowledge, most of the research regarding timetable rescheduling is devoted to incorporate new modeling characteristics, but there is limited research focusing on alternative exact algorithms for the problem (see Cacchiani et al. [1] for an updated review). Improvements in the algorithms could allow to consider larger railway networks as well as to consider certain scenarios where computing times exceed the available time. Our contribution aims to that direction and we propose two alternative MILP formulations. One of them is based on the so-called Time-Index Formulation considered in Dash et al. [2] for the Traveling Salesman Problem with Time Windows. This connection between the two problems allows to adapt and extend some of the results previoully obtained to the context of railway rescheduling. We further present preliminary computational results, including a comparison regarding the overall computing times and quality of the LP relaxations, and discuss future research directions.

## References

- 1 V. Cacchiani, D. Huisman, M. Kidd, L. Kroon, P. Toth, L. Veelenturf, and J. Wagenaar. An overview of recovery models and algorithms for real-time railway rescheduling. Transportation Research Part B: Methodological, 63(0):15–37, 2014.
- 2 S. Dash, O. Günlük, A. Lodi, and A. Tramontani. A time bucket formulation for the traveling salesman problem with time windows. INFORMS Journal on Computing, 24(1):132–147, 2012.
- 3 I. Louwerse and D. Huisman. Adjusting a railway timetable in case of partial or complete blockades. European Journal of Operational Research, 235(3):583–593, 2014.
- 4 L. P. Veelenturf, M. P. Kidd, V. Cacchiani, L. G. Kroon, and P. Toth. A Railway Timetable Rescheduling Approach for Handling Large-Scale Disruptions. Transportation Science, 2015.

# Multiple-choice problems under staircase compatibility and their applications in timetabling and routing

Andreas Bärmann

FAU Erlangen-Nürnberg, baermann@mathematik.tu-darmstadt.de

## Thorsten Gellermann

FAU Erlangen-Nürnberg, Thorsten.Gellermann@math.uni-erlangen.de

## **Frauke Liers**

FAU Erlangen-Nürnberg, Frauke.Liers@math.uni-erlangen.de

### Maximilian Merkert

FAU Erlangen-Nürnberg, Maximilian.Merkert@math.uni-erlangen.de

## Oskar Schneider

FAU Erlangen-Nürnberg, Oskar.Schneider@fau.de

We consider a multiple-choice feasibility problem where the items chosen in a solution have to fulfil a pairwise compatibility relation. This problem has a well-known specialization to project scheduling. Indeed, we motivate our work with the optimization of railway timetables to reduce the peak power consumption in the power supply system, where this multiple-choice problem occurs as a subproblem. However, we will also show the applicability of the problem framework to the linearization of physical constraints when optimizing the routing of natural gas through a pipeline system. The main part of the talk will be concerned with structural properties of the problem, focussing on compact totally unimodular problem formulations, which exist if the compatibility relation possesses a certain "staircase" structure. Furthermore, we show that the two application problems mentioned above can be solved in much shorter computation time using these totally unimodular reformulations instead of using the most direct formulation one would come up with.

The formal definition of the feasibility problem considered here is given henceforth: Let S be a basic set and  $\{S_1, \ldots, S_m\}$  a partition of S into m disjoint subsets. The multiple-choice problem under compatibility constraints (MCPC) then consists in choosing exactly one element from each subset  $S_i$ , such that the selected elements are pairwise compatible, where compatibility is given by a symmetric relation  $R \subseteq (S \times S) \setminus \bigcup_{i=1}^{m} (S_i \times S_i)$ . Two elements  $s_i, s_j \in S$  are said to be compatible if and only if  $(s_i, s_j) \in R$  holds.

Now, we assume that each subset  $S_i$  is an ordered set according to a total order  $<_i$ , for which we simply write < in the following. We then call a symmetric relation R on S a *staircase relation* if the following two conditions hold:

- [1]  $(a, b_1) \in R \land (a, b_3) \in R \implies (a, b_2) \in R$  for all  $a \in S_i, b_1, b_2, b_3 \in S_j$  with  $b_1 < b_2 < b_3$ .
- $[2] (a_1, b_2) \in R \land (a_2, b_1) \in R \implies (a_1, b_1) \in R \land (a_2, b_2) \in R \text{ for all } a_1, a_2 \in S_i, b_1, b_2 \in S_j \text{ with } a_1 < a_2 \text{ and } b_1 < b_2.$

This naming is motivated by the resulting staircase form of the adjacency matrix of the corresponding compatibility graph for partitioning into two subsets.

We call the special case of (MCPC) where R is a staircase relation the *multiple-choice problem under staircase compatibility* (MCPSC). In this case, the problem allows for a totally unimodular linear programming formulation that is linear in the number of variables and constraints. Our computational results on real-world timetabling and gas routing instances show that there is great benefit when representing the inherent (MCPSC)-substructure via the totally unimodular formulations presented in our talk.

## Timetabling of bus lines through discrete event simulation

Hande Öztop

Yaşar University, hande.oztop@yasar.edu.tr

Deniz Türsel Eliiyi Yaşar University, deniz.eliiyi@yasar.edu.tr

Uğur Eliiyi

Dokuz Eylül University, ugur.eliiyi@deu.edu.tr

The transit-operation planning process includes five basic activities, usually performed in sequence: network route design, timetable planning, vehicle scheduling, crew scheduling and crew rostering. In this study, the timetable planning stage of this process is studied for the transfer center of a public transportation authority in Izmir, Turkey. The transfer center has been selected for study based on its high passenger density due to closeness to two universities, two hospitals and a metro station. Currently, there are 22 bus lines at this transfer center, carrying around 60,000 passengers per day. The timetabling activity establishes alternative frequencies and timetables in order to meet the variations in public demand due to changes in the transportation needs of the community. The aim is to find the headways that optimize several objectives such as the minimization of the waiting time of passengers, balancing vehicle utilizations and minimizing the resource costs, where the headway is defined as the period between the departure times of two consecutive vehicles. Currently, the timetabling of the bus lines at the transfer center is done manually by the planning department, causing long waiting times of passengers at some of the bus stops and imbalanced vehicle utilizations due to ineffective planning. A discrete event simulation model is developed for the given bus lines in order to determine the best timetabling scenarios by evaluating the performance of different alternatives. As the demand varies during different time intervals, a working day is divided into time periods and the input analysis is made accordingly. Three main events are defined for the simulation system considering these time periods: passenger arrivals for generating passengers according to bus stop and time period, bus arrivals and departures for boarding/alighting activities at each bus stop, and the bus departures at each line terminus. The events are designed in a generic way in order to facilitate the changes in input parameters easily. The proposed simulation model is first tested on a pilot bus line of the aforementioned transfer center that has a high passenger density, and the numerical results are obtained. After verification and validation, other bus lines of the transfer center are added to the analysis and the whole system is simulated. Timetabling of the bus lines are optimized through the evaluation of several scenarios, and the results are reported.

## Lateness minimization for pairwise connection restoration problems

Igor Averbakh

University of Toronto Scarborough, averbakh@utsc.utoronto.ca

Jorge Pereira

Universidad Adolfo Ibanez, jorge.pereira@uai.cl

Suppose that a transportation network is damaged due to an extreme event (disaster) which destroyed its connectivity. At the first stage of network restoration, the connectivity needs to be restored. Restoring connectivity between some pairs of vertices may be more urgent than for others. We consider the setting where for all damaged edges, the required restoration effort is known, and resources are limited and fixed, which defines a constant speed of restoration activities. For each pair of vertices, a due date for restoring the connectivity of the pair is given. It is required to find an optimal schedule of restoration activities that minimizes the maximum lateness of pairs of vertices. We assume that transportation times of construction crews are negligible with respect to restoration times, and construction crews can access any part of the network at any time. We discuss complexity of the problem, its structural properties, mixed-integer linear programming formulation, and lower bounds on the optimal objective function value based on combinatorial structure of the problem. We present a branch-and-bound exact algorithm and discuss results of computational experiments.

## **Optimization of multitask radars**

Fouad Ben Abdelaziz

Neoma Business School, Fouad.BEN.ABDELAZIZ@neoma-bs.fr

#### Hasan Mir

American University of Sharjah

A radar task is an event on the radar timeline that is used to sense a new target or update the information about a previously detected target. A radar sensor performs a variety of different tasks that need to be completed before a certain time horizon. The efficient utilization of the radar timeline is thus a critical issue in the operation of modern radar systems. This paper develops a generalized framework for the radar task scheduling problem in order to allow for scheduling flexibility and the ability to handle multiple tasks using a single radar. An exact optimization model to schedule the tasks is proposed along with an efficient heuristic scheduling method. Computational results are presented to assess the behavior of the proposed method.

# Strengthened time-indexed formulations for airport runway scheduling

Pasquale Avella

DING - Università del Sannio, avella@unisannio.it

Maurizio Boccia DING - Università del Sannio, maboccia@unisannio.it

> Carlo Mannino Sintef ict, carlo.mannino@sintef.no

> > Igor Vasilyev

Matrosov Institute for System Dynamics and Control Theory of Siberian Branch of Russian Academy of Sciences, vil@icc.ru

Air traffic management typically consists of the coordinated solution of three distinct problems: The Departure Management Problem (DMAN), the Arrival Management Problem (AMAN) and the Surface Management Problem (SMAN). The DMAN problem decides take-off times for each departing flights, whereas AMAN focuses on landing times of arrival flights. Finally, the SMAN Problem decides how arriving and departing airplanes moves in the airdrome, given the established take-off times and expected landing times. Even though in principle the three problems are tightly connected and should be solved jointly, it is common practice of the airport management to handle them independently and to separate holding areas as buers between different problems. We focus on AMAN, DMAN and their integration, ADMAN (Arrival and Departure MANagement). The most common practical solution approach adopted by air traffic controllers to solve AMAN and DMAN is the simple First-Come-First-Serve rule. A very active line of research is that of studying algorithms to automatize the scheduling process and able to provide good quality solutions minimizing the total deviation from the expected departure and arrival times. We present a time-indexed formulation, exploiting the relation with a classical scheduling problem, namely the single machine with sequence de-pendent setup times. Starting from a time-indexed formulation recently proposed for single machine scheduling problems with sequence dependent setup times and re- lease dates, we present a compact reformulation based on new families of clique inequalities, leading to significantly better lower hounds. We report on preliminary computational results on real and realistic instances, validat- ing the effectiveness of the proposed approach.

## Synchronous flow shop problems with dominating machines

Sigrid Knust

University of Osnabrueck, sigrid@informatik.uni-osnabrueck.de

## Stefan Waldherr

University of Osnabrueck, stefan.waldherr@uni-osnabrueck.de

A synchronous flow shop is a variant of a non-preemptive permutation flow shop where transfers of jobs from one machine to the next take place at the same time. The processing is organized in synchronized cycles which means that in a cycle all current jobs start at the same time on the corresponding machines. Then all jobs are processed and have to wait until the last one is finished. Afterwards, all jobs are moved to the next machine simultaneously. As a consequence, the processing time of a cycle is determined by the maximum processing time of the operations contained in it. Furthermore, only permutation schedules are feasible, i.e. the jobs have to be processed in the same order on all machines. The goal is to find a permutation of the jobs such that the makespan is minimized. Motivated by a practical application we investigate special cases where the processing times of the cycles are determined by a subset of so-called dominating machines. Besides complexity results we present exact MIP formulations and heuristic solution algorithms.

# Network flow precedence based formulations for the asymmetric traveling salesman problem with precedence constraints

Luis Gouveia

University of Lisbon, legouveia@fc.ul.pt

Pierre Pesneau

University of Bordeaux, pierre.pesneau@math.u-bordeaux.fr

### Mario Ruthmair

University of Vienna, mario.ruthmair@univie.ac.at

## Daniel Santos

University of Lisbon, d.r.santos@outlook.com

Given a directed graph G = (V, A), a cost function c associated with the arcs of A, and a set of precedence constraints  $B \subset V \times V$ , the Precedence Constrained Asymmetric Traveling Salesman Problem (PCATSP) seeks for a minimum cost Hamiltonian circuit, starting at node 1, and such that for each  $(i, j) \in B$ , the node i is visited before node j.

There are many ways of modelling the ATSP and several for the PCATSP. In this talk we present new formulations for the two problems that can be viewed as resulting from combining precedence variable based formulations, with network flow based formulations. As suggested in [1], the former class of formulations permits to integrate linear ordering constraints.

The motivating formulation for this work is a complicated and "ugly" formulation that results from the separation of generalized subtour elimination constraints presented in [2] (see also [1]).

This so called "ugly" formulation exhibits, however, one interesting feature, namely the "disjoint subpaths" property that is further explored to create more complicated formulations that combine two (or three) "disjoint path" network flow based formulations and have a stronger linear programming bound. Some of these stronger formulations are related to the ones presented for the PCATSP in [3] and can be viewed as generalizations in the space of the precedence based variables.

Several sets of projected inequalities in the space of the arc and precedence variables and in the spirit of many presented in [1] are obtained by projection from these network flow based formulations.

Computational results will be given for the ATSP and PCATSP to evaluate the quality of the new models and inequalities.

## References

- L. Gouveia and P. Pesneau. On extended formulations for the precedence constrainted asymmetric traveling salesman problem. Networks, 48(2):77–89, 2006.
- [2] L. Gouveia and J. M. Pires. The asymmetric travelling salesman problem: On generalizations of disaggregated Miller-Tucker-Zemlin constraints. Discrete Applied Mathematics, 112:129–145, 2001.
- [3] A. N. Letchford and J.-J. Salazar-González. Stronger multi-commodity flow formulations of the (capacitated) sequential ordering problem. European Journal of Operational Research, 251(1):74–84, 2016.

## A set cover approach for the double traveling salesman problem with multiple stacks

Michele Barbato

Université Paris 13, Sorbonne Paris Cité, LIPN, CNRS (UMR 7030), barbato@lipn.fr

Roland Grappe

Université Paris 13, Sorbonne Paris Cité, LIPN, CNRS (UMR 7030)

Mathieu Lacroix

Université Paris 13, Sorbonne Paris Cité, LIPN, CNRS (UMR 7030)

Roberto Wolfler Calvo

Université Paris 13, Sorbonne Paris Cité, LIPN, CNRS (UMR 7030)

In the double TSP with multiple stacks, a vehicle with several stacks performs a Hamiltonian circuit to pick up some items and stores them in its stacks. It then delivers every item by performing another Hamiltonian circuit while satisfying the last-in-first-out policy of its stacks. The consistency requirements ensuring that the pickup and delivery circuits can be performed by the vehicle is the major difficulty of the problem. These consistency requirements correspond, from a polyhedral standpoint, to a set covering polytope. We show that when the vehicle has two stacks this polytope is obtained from the description of a vertex cover polytope. We use these results to develop a branch-and-cut algorithm with inequalities derived from the inequalities of the vertex cover polytope.

## The parity hamiltonian cycle problem in directed graphs

Hiroshi Nishiyama

Kyushu University, nyama@tcslab.csce.kyushu-u.ac.jp

Yukiko Yamauchi *Kyushu University*, yamauchi@inf.kyushu-u.ac.jp

Shuji Kijima *Kyushu University*, kijima@inf.kyushu-u.ac.jp

## Masafumi Yamashita

Kyushu University, mak@inf.kyushu-u.ac.jp

This paper investigates a variant of the Hamiltonian cycle problem, the parity Hamiltonian cycle (PHC) problem: a PHC in a directed graph is a closed walk (possibly using an arc more than once) which visits every vertex odd number of times. Nishiyama et al. (2015) investigated the PHC problem in undirected graphs, and showed that a connected undirected graph has a PHC if and only if it is even order or nonbipartite. This paper gives a characterization when a directed graph is solved in polynomial time. The characterization, completely dissimilar to the undirected case, is described by a linear systems over GF(2).

# Flow and layered graph models for the black-and-white traveling salesman problem

Luis Gouveia University of Lisbon, legouveia@fc.ul.pt

Mario Ruthmair University of Vienna, mario.ruthmair@univie.ac.at

#### Markus Leitner

University of Vienna, markus.leitner@univie.ac.at

We consider a variant of the traveling salesman problem which arises in telecommunication networks and airline maintenance operations. The blackand-white traveling salesman problem asks for a Hamiltonian tour with minimal total distance subject to additional constraints: The node set is partitioned into white and black nodes and the number of white nodes between two consecutive black nodes in a feasible tour is limited (cardinality constraint). Additionally, we have to ensure a distance constraint on the path between two consecutive black nodes.

Ghiani et al. (2006) propose a branch-and-cut algorithm based on a mixed integer linear programming formulation in the natural variable space which works well for loosely constrained instances. Muter (2015) uses a branch-andprice approach based on a model with an exponential number of variables corresponding to black-to-black paths. Here, tightly-constrained instances are easier to solve since the set of feasible paths generated in the pricing subproblem is usually smaller.

Following the motivation given by the applications in telecommunications we model the problem as a combination of two subproblems, each one associated to a different layer. In the upper layer we model a Hamiltonian tour only on the set of black nodes. In the lower layer we model a Hamiltonian tour on the whole node set. Both layers are then combined with adequate coupling constraints.

Second, we present extended formulations in which the cardinality and distance constraints are modeled on appropriate layered graphs. We also combine the two layered graphs in a single three-dimensional layered graph ensuring both constraints at once.

The models are solved by branch-and-cut including extensive preprocessing to reduce the size of the layered graphs and heuristics to obtain good primal bounds for pruning the branch-and-bound tree. Preliminary results indicate that our solution methods are competitive to the existing approaches and especially perform well in case of tight constraints.

## Towards an accurate solution of wireless network design problems

Fabio D'Andreagiovanni

Zuse Institute Berlin (ZIB), d.andreagiovanni@zib.de

Ambros Gleixner Zuse Institute Berlin (ZIB), gleixner@zib.de

The optimal design of wireless networks has been widely studied in the literature and many optimization models have been proposed over the years. However, most models directly include the signal-to-interference ratios representing service coverage conditions. This leads to mixed-integer linear programs with constraint matrices containing tiny coefficients that vary widely in their order of magnitude. These formulations are known to be challenging even for state-of-the-art solvers: the standard numerical precision supported by these solvers is usually not sufficient to reliably guarantee feasible solutions. Service coverage errors are thus commonly present. Though these numerical issues are known and become evident even for small-sized instances. just a very limited number of papers has tried to tackle them, by mainly investigating alternative non-compact formulations in which the sources of numerical instabilities are eliminated. In this work, we explore a new approach by investigating how recent advances in exact solution algorithms for linear and mixed-integer programs over the rational numbers can be applied to analyze and tackle the numerical difficulties arising in wireless network design models.

# Two-fold circle-covering of the plane under congruent Voronoi polygon conditions

### Jingchao Chen

Department of Communication, Donghua University, chen-jc@dhu.edu.cn

The k-coverage problem is to find the minimum number of disks such that each point in a given plane is covered by at least k disks. Under unit disk condition, when k = 1, this problem has been solved by Kershner in 1939. However, when k > 1, it becomes extremely difficult. One tried to tackle this problem with different restrictions. In this paper, we restrict ourself to congruent Voronoi polygon, and prove the minimum density of the twocoverage with such a restriction. Our proof is simpler and more rigorous than that given recently by Yun et al.

# Reducing the clique and chromatic number via edge contractions and vertex deletions

Daniel Paulusma

School of Engineering and Computing Sciences, Durham University, daniel.paulusma@durham.ac.uk

> Christophe Picouleau CEDRIC-CNAM, chp@cnam.fr

> > Bernard Ries

Université Paris Dauphine, bernard.ries@dauphine.fr

We consider the following problem:

Can a certain graph parameter of some given graph G be reduced by at least d, for some integer d,via at most k graph operations from some specified set S, for some given integer k?

As graph parameters we take the chromatic number and the clique number. We let the set S consist of either an edge contraction or a vertex deletion. As all these problems are NP-complete for general graphs even if d is fixed, we restrict the input graph G to some special graph class.

We continue a line of research that considers these problems for subclasses of perfect graphs, but our main results are full classifications, from a computational complexity point of view, for graph classes characterized by forbidding a single induced connected subgraph H.

# The asymmetric VPN tree problem: formulation and polyhedral investigation

Ibrahima Diarrassouba Université du Havre, diarrasi@univ-lehavre.fr

Pedro Henrique Liguori Université Paris Dauphine, phliguori@gmail.com

A. Ridha Mahjoub Université Paris Dauphine, mahjoub@lamsade.dauphine.fr

In this paper, we consider the VPN tree problem in the asymmetric case. We first investigate the computational complexity of the problem and show that it is NP-Hard. We also give an integer programming formulation for the problem, which, contrarily to the paper of Kumar et al. 2002, allows finding an optimal solution of the problem by solving a unique integer program.

# The set covering polyhedron of circular matrices: minor vs. row family inequalities

Silvia Bianchi

Universidad Nacional de Rosario, sbianchi@fceia.unr.edu.ar

Graciela Nasini

Universidad Nacional de Rosario - CONICET, nasini@fceia.unr.edu.ar

Paola Tolomei Universidad Nacional de Rosario - CONICET, ptolomei@fceia.unr.edu.ar

#### Luis Miguel Torres

ModeMat - Escuela Politécnica Nacional, Quito, luis.torres@epn.edu.ec

In the context of the study of the set covering polyhedron of circular matrices, we study the relationship between row family inequalities and a previously proposed class termed minor inequalities [1]. Recently, following the ideas for the stable set polytope on circular graphs [3], a complete linear description of this polyhedron has been provided in terms of row family inequalities [4]. In this work we prove that for the particular subclass of circulant matrices, the corresponding row family inequalities are related to circulant minors. Moreover, we extend previous results on circulant matrices  $C_{sk}^k$ ,  $s \in \{2,3\}$  and  $k \ge 2$  [2], by providing a polynomial time separation algorithm for the inequalities describing the set covering polyhedron of matrices  $C_{4k}^k$ ,  $k \ge 2$ .

## References

- S. Bianchi and G. Nasini and P. Tolomei: Some advances on the set covering polyhedron of circulant matrices, Discrete Applied Mathematics Vol 166 (2014) 59–60.
- [2] S. Bianchi and G. Nasini and P. Tolomei: The minor inequalities in the description of the set covering polyhedron of circulant matrices, Math.

Meth. Oper. Res. Vol 79 (2014) 69–85.

- [3] F. Eisenbrand and G. Oriolo and G. Stauffer and P. Ventura : *The stable* set polytope of quasi-line graphs, Combinatorica Vol 28-1 (2008) 45–67.
- [4] L.M. Torres: Minor related row family inequalities for the set covering polyhedron of circulant matrices, Electronic Notes in Discrete Mathematics Vol 50 (2015) 325–330.

# A branch-and-cut approach for the minimum branch vertices spanning tree problem

Selene Silvestri University of Salerno, selene.silvestri@gmail.com

Gilbert Laporte HEC Montreal, Gilbert.Laporte@cirrelt.ca

Raffaele Cerulli University of Salerno, raffaele@unisa.it

Given a connected undirected graph G = (V, E), the Minimum Branch Vertices Problem (MBVP) asks for a spanning tree of G with the minimum number of vertices having degree greater than two in the tree. Such vertices are called branch. The problem finds application in the context of optical networks and is known to be NP-hard. We model the MBVP as an integer linear program with undirected variables, we derive valid inequalities and prove some these to be facet defining. We also develop a hybrid formulation containing undirected and directed variables. Both models are solved by branch-and- cut. Comparative computational results show the superiority of the hybrid formulation.

# The k-regular induced subgraph problem

**Torkel Andreas Haufmann** 

University of Oslo, torkelah@math.uio.no

Agostinho Agra University of Aveiro, aagra@ua.pt

Geir Dahl University of Oslo, geird@math.uio.no

## Sofia Pinheiro

University of Aveiro, spinheiro@ua.pt

An induced subgraph of a graph G consists of a subset of the vertices of G along with all edges in G having both endpoints in said subset. A k-regular graph is a graph in which every vertex has degree k. In this talk we consider the problem of finding the maximum cardinality k-regular induced subgraph of a graph.

This problem generalizes, among others, the maximal independent set problem, the induced matching problem and the maximal clique problem. It has been shown that for any k the problem is NP-hard, yet some polynomialtime computable bounds have been given in the literature, respectively based on convex quadratic programming and the spectra of adjacency, Laplacian and signless Laplacian matrices of the given graph.

We review some of the known bounds, and consider an integer linear programming formulation. A relation between a Lagrangian relaxation and a bound of Cardoso, Kaminski and Lozin is given. We also consider cases for which the maximal induced k-regular subgraph problem can be solved in polynomial time. Finally computational results are given for the integer linear program and its relaxations, and compared with the tractable bounds.

## Optimization problems with color-induced budget constraints

Corinna Gottschalk

RWTH Aachen University, gottschalk.corinna@gmail.com

Hendrik Lüthen

 $TU \ Darmstadt$ , luethen@mathematik.tu-darmstadt.de

Britta Peis

RWTH Aachen University, britta.peis@rwth-aachen.de

## Andreas Wierz

RWTH Aachen University, and reas.wierz@oms.rwth-aachen.de

Gabow and Tarjan provided a very elegant and fast algorithm for the following problem: given a matroid defined on a red-blue colored ground set, determine a basis of minimum cost among those with k red elements, or decide that no such basis exists. In this paper, we investigate possible extensions of this result from ordinary matroids to the more general notion of *poset matroids*. Poset matroids (also called *distributive supermatroids*) are defined on the collection of all ideals of an underlying partial order on the ground set. We show that the problem on general poset matroids becomes NP-hard, already if the underlying poset consists of binary trees of height two. On the positive side, we present polynomial-time algorithms for (1) integer polymatroids, i.e., the case where the poset consists of disjoint chains, and (2) the problem to determine a min-cost ideal of size l with k red elements, i.e., the uniform rank-l poset matroid, on series-parallel posets.

# Projection results for the k-partition problem

Jamie Fairbrother

Lancaster University, j.fairbrother@lancaster.ac.uk

## Adam Letchford

Lancaster University, a.n.letchford@lancaster.ac.uk

In the *k*-partition problem (*k*-PP), we are given an undirected graph G = (V, E) and a rational weight  $w_e$  for each  $e \in E$ . The task is to partition V into k or fewer subsets (called "clusters"), such that the sum of the weights of the edges that have both end-vertices in the same cluster is minimised. The *k*-PP has applications in statistical clustering, numerical linear algebra, telecommunications, VLSI layout, sports team scheduling and statistical physics (see, e.g., the references below).

Note that the k-PP is equivalent to the max-k-cut problem, in which one wishes to maximise the sum of the weights of the edges that have exactly one end-vertex in the same cluster. It is therefore  $\mathcal{NP}$ -hard in the strong sense, even when k = 2.

Chopra and Rao (1993) introduced two integer programming formulations of the k-PP. The first one, which we call the *node-and-edge* formulation, has k binary variables for each node and one binary variable for each edge. The other, which we call the *edge-only* formulation, has one binary variable for each pair of nodes.

We show that, if we take the polytopes associated with the edge-only formulation, and project them into a suitable subspace, we obtain the polytopes associated with the node-and-edge formulation. This result enables us to derive several new families of valid inequalities for the edge-only formulation, including *projected clique*, *projected odd wheel*, *projected odd bicycle wheel* and *projected 2-chorded odd cycle* inequalities. For the latter three families, we also obtain polynomial-time exact separation algorithms.

As a by-product, we obtain a new semidefinite programming (SDP) relaxation. Unlike the standard SDP relaxations, which have matrix variables of order n or order nk+1 (see the last three references below), our relaxation has a matrix variable of order n + k.

## References

- S. Chopra & M.R. Rao (1993) The partition problem. Math. Program., 59, 87-115.
- [2] A. Eisenblaetter (2002) The semidefinite relaxation of the k-partition polytope is strong. In W.J. Cook & A.S. Schulz (eds.) Integer Programming and Combinatorial Optimization IX, pp. 273-290. Lecture Notes in Computer Science, vol. 2337. Berlin: Springer.
- [3] B. Ghaddar, M.F. Anjos & F. Liers (2011) A branch-and-cut algorithm based on semidefinite programming for the minimum k-partition problem. Ann. Oper. Res., 188, 155-174.
- [4] F. Rendl (2012) Semidefinite relaxations for partitioning, assignment and ordering problems. 4OR, 10, 321-346.

# Improved compact formulations for a wide class of graph partitioning problems in sparse graphs

Dang Phuong Nguyen CEA-LIST, dang-phuong.nguyen@polytechnique.edu

Michel Minoux Sorbonne Universités, UPMC Univ Paris 06, UMR 7606, LIP6, michel.minoux@lip6.fr

> **Thanh Hai Nguyen** *CEA-LIST*, thanhhai.nguyen@cea.fr

Viet Hung Nguyen Sorbonne Universités, UPMC Univ Paris 06, UMR 7606, LIP6, hung.nguyen@lip6.fr

> Renaud Sirdey CEA-LIST, renaud.sirdey@cea.fr

The graph partitioning problem with set constraints that we consider can be generically defined as follows. Given an undirected connected graph G = (V, E) where  $V = \{1, \ldots, n\}, |E| = m$  and a length  $l_e \in \mathbb{Z}_+$  is associated with each edge  $e \in E$ , find a partition of V into disjoint sets (or clusters) such that:

- every cluster  $C \subset V$  satisfies a constraint of the form  $\mathcal{G}(y^C) \leq 0$  where  $y^C$  is the incidence vector of C in  $\{0,1\}^n$  and  $\mathcal{G}: \{0,1\}^n \to \mathbb{R}$  is a given monotone nondecreasing pseudoboolean function.
- the sum of the lengths of the edges having end-nodes in different clusters is minimized.

The above class of problems (denoted GPP-SC) is fairly general, and includes several special cases of practical interest in particular: the graph partitioning problem under knapsack constraints [Garey & Johnson, W. H. Freeman & Co, 1979] and the graph partitioning problem under capacity constraints [Bonami et al., LNCS.7422, 2012]. The node partitions of a graph partitioning problem are often modelled by Node-Node model that is based on the choice of decision variables  $x_{ij}$  which are equal to 1 iff node *i* and node *j* are not in the same cluster. The result is a 0/1 linear program and makes use of  $O(n^2)$  variables and  $O(n^3)$  triangle inequalities :

$$\begin{cases} x_{uv} + x_{uw} \ge x_{vw} & \forall u, v, w \in V, u < v < w \\ x_{uv} + x_{vw} \ge x_{uw} & \forall u, v, w \in V, u < v < w \\ x_{vw} + x_{uw} \ge x_{uv} & \forall u, v, w \in V, u < v < w \\ x_{uv} \in \{0, 1\} & \forall u, v \in V, u < v \end{cases}$$

The number of triangle inequalities quickly becomes extremely large as n increases and even the linear relaxation turns out to be difficult to solve. Some authors have tried to overcome this difficulty by dualizing all or only a subset of the triangle inequalities via a Lagrangian approach [Frangioni et al., Math.Prog.104, 2005]. But beyond that, it would be interesting to be able to reduce intrinsically the number of triangle inequalities without weakening the linear relaxation. We will show that with only O(nm) triangle inequalities, instead of  $O(n^3)$ , we can obtain an equivalent formulation, not only for the Node-Node model for GPP-SC, but also for its linear programming relaxation. Obviously, such a reduction opens the way to considerable improvement in case of sparse graphs where  $m \ll \frac{n(n-1)}{2}$ .

Numerical results have shown that solution times are reduced drastically from 3 to 50 times with our reduced formulation. There are instances for which the LP solver is not even capable of solving the continuous relaxation with the classical formulation but succeeds at finding optimal integer solutions with our reduced formulation.

## Balanced partition of a graph for football team realignment in Ecuador

Diego Recalde

Escuela Politécnica Nacional, diego.recalde@epn.edu.ec

Daniel Severín

Universidad Nacional de Rosario, daniel@fceia.unr.edu.ar

**Ramiro Torres** 

Escuela Politécnica Nacional, ramiro.torres@epn.edu.ec

### Polo Vaca

Escuela Politécnica Nacional, polo.vaca@epn.edu.ec

In the second category of the Ecuadorian football league, a set of football teams must be grouped in k geographical zones according to some regulations, where the total distance of the road trips that all teams must travel to play a Double Round Robin Tournament in each zone is minimized. This problem can be modeled as a k-clique partitioning problem with constraints on the sizes and weights of the cliques. An integer programming formulation and a heuristic approach were developed to provide a solution to the problem which has been implemented in the 2015 edition of this football championship.

# Uniqueness of equilibria in atomic splittable polymatroid congestion games

Veerle Timmermans

Maastricht University, v.timmermans@maastrichtuniversity.nl

#### **Tobias Harks**

University of Augsburg, tobias.harks@math.uni-augsburg.de

We study uniqueness of Nash equilibria in atomic splittable congestion games and derive a uniqueness result based on polymatroid theory: when the strategy space of every player is a bidirectional flow polymatroid, then equilibria are unique. Bidirectional flow polymatroids are introduced as a subclass of polymatroids possessing certain exchange properties. We show that important cases such as base orderable matroids can be recovered as a special case of bidirectional flow polymatroids. On the other hand we show that matroidal set systems are in some sense necessary to guarantee uniqueness of equilibria: for every atomic splittable congestion game with at least three players and non-matroidal set systems per player, there is an isomorphic game having multiple equilibria. Our results leave a gap between base orderable matroids and general matroids for which we do not know whether equilibria are unique.

# A set partitioning reformulation for a multi-attribute surgery planning problem with uncertain surgery durations

Mahdi Noorizadegan

Amirkabir University of Technology, m.noorizadegan@gmail.com

## Abbas Seifi

Amirkabir University of Technology, aseifi@aut.ac.ir

Operating rooms accounts for more than 40% of the total revenue and the total costs of hospitals. Therefore, an efficient planning for surgeries has a significant impact of hospitals' revenue and cost. There is an increasing interest from the operations research society to solve various types of surgery planing problems (see [1] and [2]).

This paper investigates a multi-attribute surgery planning problem with uncertain surgery durations. The problem is characterized by a set of identical operating rooms (ORs), a set of surgeries, a planning horizon, a set of surgeons, uncertain surgery durations, and surgery dependent turn-over times. The decisions to be made before the realization of random surgery durations include which ORs to open, assignments of surgery-surgeon pairs to feasible/prioritized dates and open ORs, and the sequence of surgeries. The objective function minimizes the opening cost and penalized turn-over times.

A set partitioning reformulation is proposed to model the above stochastic surgery planning problem where probabilistic constraints are used to deal with the uncertain durations. Two additional realistic constraints are the limitation on the number of ORs opened in each period (e.g., day), and the fact that a surgeon can not be assigned to two surgeries of his/her surgery list at the same time.

We use a branch-and-price framework to solve the resulting problem. Although the resulting pricing problem is the decomposition of the problem by period and by OR, it is still difficult to solve. We exploit the structure of the pricing problem and reformulate it by a modified shortest path problem. We conduct a comprehensive computational experiments to test the efficiency of our proposed algorithm and the solution quality. The results indicates that the proposed method dominates the canonical integer chance constrained programming model (solved by branch-and-cut methods) in terms of the solution quality and computational efficiency. We also investigate the surgeon waiting time for the cases when only OR opening is taken into account versus the cases when turn-over times is penalized. Furthermore, an extensive simulation experiment is carried out to compare some variants of the underlying problem.

## References

- 1 N. K. Freeman, S. H. Melouk, and J. Mittenthal, A scenario-based approach for operating theater scheduling under uncertainty, Manufacturing & Service Operations Management, Articles in Advance, (2015), pp. 1—17.
- 2 O. V. Shylo, O. A. Prokopyev, and A. J. Schaefer, Stochastic operating room scheduling for high-volume specialties under block booking, IN-FORMS Journal on Computing, 25 (2013), pp. 682–692.
### A compact representation for minimizers of k-submodular function

Hiroshi Hirai

The university of Tokyo, hirai@mist.i.u-tokyo.ac.jp

Taihei Oki

The university of Tokyo, taihei\_oki@me2.mist.i.u-tokyo.ac.jp

k-submodular functions were introduced by Huber and Kolmogorov as a generalization of bisubmodular functions. This paper introduces a compact representation of minimizers of k-submodular functions by posets with inconsistent pairs (PIPs). We give algorithms to construct the PIP representing the minimizer set of a k-submodular function f for three cases: (i) a minimizing oracle of f is given, (ii) f is network-representable, and (iii) fis a relaxation of the multiway cut problem. We completely characterize the class of PIPs (elementary PIPs) corresponding to minimizers of k-submodular functions. By using this characterization, we provide an efficient algorithm to enumerate all maximal minimizers of a k-submodular function. Our results are applied to obtain all maximal persistent assignments in labeling problems arising from computer vision.

# Optimisation of training algorithm of temporal neuro-fuzzy system for fault prognostic in manufacturing system

Mahdaoui Rafik

ICOSI khenchela University, mehdaoui.rafik@yahoo.fr

The purpose of this paper is to propose a Temporal Neuro-Fuzzy System to solve the fault prognostic of Manufacturing System. To solve this problem, a new structure of Neuro-fuzzy system and a training algorithm is proposed. Fault prognosis techniques have been proposed to improve the performance of predictions algorithms.

# New very-large scale neighbourhoods for a family of partitioning problems

Anh Vu

Lancaster University Management School, a.vu@lancaster.ac.uk

#### Adam Letchford

Lancaster University Management School, a.n.letchford@lancaster.ac.uk

Heuristics based on neighbourhood search have proven to be useful for many combinatorial optimisation problems. One stream of research has focused on so-called very large scale neighbourhoods (VLSNs), which are of exponential size, yet can be searched efficiently (either exactly or heuristically). For details, see the surveys by Ahuja et al. (2002) and Altner et al. (2014).

We are interested in VLSNs for a broad family of partitioning problems, in which the task is to partition a ground set into subsets, in order to maximise or minimise a cost function that depends only on the subsets. This family of problems includes several important routing, scheduling, packing, clustering and network design problems. For these partitioning problems, three VLSNs have been proposed in the literature. For brevity, we call them "cyclic exchange", "match" and "split-and-assign".

We start by analysing the three VLSNs mentioned. It turns out that the cyclic exchange neighbourhood has the largest cardinality, and the split-and-assign neighbourhood has the smallest. On the other hand, it is NP-hard to search the cyclic exchange and matching neighbourhoods exactly, whereas the split-and-assign neighbourhood can be searched exactly in polynomial time.

We then propose three more VLSNs of our own, called "ordered path exchange", "split-and-match" and "simple path". All three can be searched exactly in polynomial time. The ordered path exchange neighbourhood is a modified version of the cyclic exchange neighbourhood, and the split-andmatch neighbourhood is an improved version of the split-and-assign neighbourhood. The simple path neighbourhood is completely new. It turns out that the split-and-match neighbourhood is the largest known VLSN that can be searched exactly in polynomial time. It has cardinality  $\Theta(e^{\sqrt{n} {n \choose e}^{n/2}})$  and can be searched in  $\mathcal{O}(n^3)$  time.

We intend in future to apply our VLSNs to the clique partitioning problem (see Groetschel and Wakabayashi, 1989), which has applications in, e.g., statistical clustering, task allocation, group technology and microarray data analysis.

This talk is based on joint work with my PhD supervisor, Adam N. Letchford.

#### References

- R.K. Ahuja, O. Ergun. J.B. Orlin & A.P. Punnen (2002) A survey of very large-scale neighborhood search techniques. Discr. Appl. Math., 123, 75-102.
- [2] D.S. Altner, R.K. Ahuja, O. Ergun & J.B. Orlin (2014) Very large-scale neighborhood search. In E. Burke & G. Kendall (eds.) Search Methodologies: Introductory Tutorials in Optimization and Decision Support Techniques, pp. 339-367. Springer US.
- [3] M. Groetschel & Y. Wakabayashi (1989) A cutting plane algorithm for a clustering problem. Math. Program., 45, 59-96.

# Aggregation technique applied to a clustering problem

#### Jeremy Guillot

University of Bordeaux, jeremy.guillot@math.u-bordeaux.fr

#### Francois Clautiaux

University of Bordeaux, francois.clautiaux@math.u-bordeaux.fr

#### Pierre Pesneau

University of Bordeaux, pierre.pesneau@math.u-bordeaux.fr

In this talk, we consider the problem of covering an urban area with sectors under additional constraints. We will present how we adapt this method to our column generation algorithm. In particular, we will focus on the problem of disaggregating the dual solution returned by the aggregated master problem.

## A tight relaxation of the energy optimization problem

Libor Bukata

The Czech Technical University in Prague, bukatlib@fel.cvut.cz

Přemysl Šůcha

The Czech Technical University in Prague, suchap@fel.cvut.cz

#### Zdeněk Hanzálek

The Czech Technical University in Prague, hanzalek@fel.cvut.cz

Robotic cells, ubiquitous in automobile industry, consume a substantial part of energy required for manufacturing, therefore there is a demand for an energy optimization that could reduce expenses and improve the global environment. Exact algorithms for such a non-linear optimization require a good problem relaxation to ensure effective pruning of the solution space. Therefore, our goal is to propose a problem relaxation, i.e. a lower bound on energy consumption, that is solvable fast, is a good approximation of the original problem (see [1] for its mathematical formulation) in terms of the estimation of the energy consumption, and is also sensitive to decisions made by an exact algorithm. The relaxation is defined for each robot as follows:

$$\min_{\boldsymbol{d}} \quad \sum_{\substack{j=1\\j=1}}^{n} g_j(d_j) \\ \boldsymbol{1}^T \boldsymbol{d} = \mathbf{K} \\ \underline{\boldsymbol{d}} \leq \boldsymbol{d} \leq \overline{\boldsymbol{d}}, \quad \boldsymbol{d} \in \mathbb{R}^n$$

where  $g_j \colon \mathbb{R} \to \mathbb{R}$  are continuous convex (not necessarily strictly) functions of class  $C^0$ , K is a problem constant (cycle time), and d is a vector of n bounded variables (durations). Each convex function  $g_j$  maps the duration (e.g. of a robot movement or waiting) to energy consumption and its shape is determined by current decisions of an exact algorithm. The time relations between robots are reflected in bounds  $\underline{d}$  and  $\overline{d}$ . To solve this problem, namely the Continuous Non-linear Resource Allocation Problem, Lagrange multiplier or pegging algorithms are often employed. Nevertheless, these algorithms are not directly usable in our case because of the form of  $g_j$  functions that are

115

not strictly convex and differentiable, therefore, we adapted Dafermos and Sparrow's algorithm that was originally designed for the Traffic Assignment Problem [2]. The core of our algorithm is an equilibrium operator that takes a pair of convex functions  $(g_i, g_j)$  and tries to increase/decrease  $d_i$  and decrease/increase  $d_i$  such that the criterion is improved and the equality and bounding constraints are not violated. This iterative process continues until there is no equilibrium operator to be applied, in that case an optimal solution has been found because of the problem convexity. Our algorithm was compared with a reference Linear Programming formulation where the nonlinear convex functions were substituted by piece-wise linear functions. The results indicate that our algorithm is about 7 to 30 times faster (depending on n and sampling) than a general Gurobi solver for piece-wise linear convex functions. So, in summary, the first contribution is an efficient method for solving problem relaxations that are directly usable in exact algorithms. and the second one is a modeling approach defining how to shape the convex functions based on decisions made by an exact algorithm.

#### References

- 1 L. Bukata, P. Sucha, Z. Hanzalek, and P. Burget. Mathematical formulation of the original problem. http://ctu-iig.github.io/EnergyOptimizatorOfRoboticCells/math\_ form.html.
- 2 S. C. Dafermos and F. T. Sparrow. The Traffic Assignment Problem for a General Network. J. Res. Nat. Bur. Stand. (U.S.), 73B(2):91 – 118, April-June 1969.

# Hop-constrained electricity network design problems

Jérôme De Boeck *GOM* - *ULB*, jeromedeboeck@gmail.com

Bernard Fortz *GOM* - *ULB*, bernard.fortz@ulb.ac.be

Electricity distribution networks can be subject to constraints that limit the number of intermediate nodes between some elements of the network (hop-constraints) in order to limit power losses. Representing hop-constraints in mixed integer optimization models can lead to an exponential number of constraints. We propose a compact extended integer model based on layered graphs modelling distances in a connected network. The resulting number of constraints depends linearly on the number of customers, feeders and maximum tolerated hops.

This model allows to represent an approximation of power losses which can significantly improve the solving time of a problem due to broken symetries in the solution space and the choice of the a hop-constraints that can be more accurate.

We study our model on the hop-constrained minimum margin problem in an electricity network. Given a set of feeders with a given capacity, the problem consists in designing a connected electricity distribution network, and to assign customers to feeders so as to maximize the minimum power margin over the feeders. Moreover, we impose a limit on the length of the path between each customer and its feeder (hop-constraints), in order to avoid too important power losses due to transportation.

Numerical results of our model are compared with results of a cuttingplane algorithm of A. Rossi[1]. The solving times variations due to the loss function used and the value of the hop-constraint will be presented.

#### References

[1] A.Rossi.Connectivity-and-hop-constrained design of electricity distribution networks, European Journal of Operational Research 218 (2012) 48-57.

## A dynamic programming approach to design a robust renewable energy park

Alain Billionnet

ENSIIE, alain.billionnet@ensiie.fr

Marie-Christine Costa

Ensta-Paristech / CNAM, marie-christine.costa@ensta-paristech.fr

#### **Pierre-Louis** Poirion

Ensta-Paristech / CNAM, poirion@lix.polytechnique.fr

In this paper we propose a two-stage robust approach based on dynamic programming to design a stand-alone hybrid energy park. To compensate for a possible lack of energy from wind turbines, solar photovoltaic panels and batteries, an auxiliary fuel generator guarantees to meet the demand in every case but its use induces important costs. We seek to determine the energy park that generates a minimum total cost in case a worst scenario occurs. We propose a polynomial time dynamic programming algorithm for the recourse problem and we use a constraint generation algorithm to solve the global robust problem. We report computational experiments on instances constructed from real data, that show the efficiency of the proposed approach.

# Scheduling personnel retraining: a column generation approach

Oliver Czibula

University of Technology Sydney, oliver.g.czibula@student.uts.edu.au

#### Hanyu Gu

University of Technology Sydney, hanyu.gu@uts.edu.au

#### Yakov Zinder

University of Technology Sydney, yakov.zinder@uts.edu.au

Many organisations must periodically provide training for their staff. One such example is Ausgrid, Australia's largest electricity distributor. Due to the hazardous nature of the work, Australian law requires all people working on or near the electricity network to undergo safety and technical training to ensure works are carried out safely and correctly. Ausgrid provides much of this training to thousands of its employees, contractors, and also to third parties. Most training courses have a limited period of validity, after which it expires. Workers may only perform works for which they are up-to-date with all required qualifications.

Due to the high degree of technical and professional diversity in the workforce, it is desirable to form classes composed of similar types of students. This enables training to be better tailored to the specific needs of the group. Due to the scarcity of training resources and cost of running additional training sessions, it is often not practical to run completely segregated classes, and some blending of student types is therefore required.

There are penalties for pairing particular student types together in classes, and also for assigning students to particular classes. The considered problem involves assigning students to classes, where each class has between a minimum and maximum number of students and of student types, such that the penalty of assigning two student types to the same class, and the penalty assigning students to particular classes, is minimised.

The paper presents linear and nonlinear mathematical programming formulations for the problem. A column generation procedure is presented, together with a set-covering (master) formulation. Three approaches are proposed, each based on the ideas of column generation:

- [1] Straightforward application of the column generation procedure;
- [2] Column generation, involving the linear programming relaxation of the master problem, with subsequent variable fixing for various columns; and
- [3] Column generation, involving the linear programming relaxation of the master problem, resulting in additional restrictions on the patterns of assignment which can be used.

The three solution approaches are compared by means of computational experiments, using a number of test cases that were randomly generated with similar characteristics to real-world cases from Ausgrid.

# Integrated production scheduling and delivery routing: complexity results and column generation

**Azeddine Cheref** *Université de tours/ LAAS-CNRS*, acheref@laas.fr

> Christian Artigues CNRS, LAAS, artigues@laas.fr

Jean-Charles Billaut Université de tours, jean-charles.billaut@univ-tours.fr

> Sandra Ulrich Ngueveu CNRS, LAAS, ngueveu@laas.fr

In this paper, we study an integrated production scheduling and delivery routing problem. The manufacturer has to schedule a set of jobs on a single machine without preemption and to deliver them to multiple customers. A single vehicle with limited capacity is used for the delivery. For each job are associated: a processing time, a size and a specific customer location. The problem consists then to determine the production sequence, constitutes batches and to find the best delivery sequence for each batch. The objectives of the proposed problems are to find a coordinated production and a delivery schedule that minimises the total completion time (makespan) or the sum of the delivery times of the products. We present complexity results for particular cases and a column generation scheme for both criteria. Some computational results show the good performances of the method.

## Optimization models for multi-period railway rolling stock assignment

Susumu Morito

Waseda University, morito@waseda.jp

Yuho Takehi Waseda University, yuhotakehi@akane.waseda.jp

> Jun Imaizumi Toyo University, jun@toyo.jp

Takayuki Shiina Chiba Institute of Technology, shiina.takayuki@it-chiba.ac.jp

It is necessary for railway companies to construct daily schedules of assigning rolling stocks to utilization paths. A utilization path consists of a series of trains that a particular rolling stock performs in a day. A mixed integer programming model based on Lai et al. is presented and is shown that straightforward applications of the model result in too much computational time and also inappropriate assignment schedules due to end effects. We show that the model can be modified to alleviate these difficulties, and also show that the repeated applications of the optimization model in the rolling horizon allow to generate a feasible assignment schedule for a longer period of time thus indicating the feasibility of the optimization approach.

# Multiple disjoint spanning trees for bi-rotator graphs

Cheng-Jhe Lee

National Taiwan University of Science and Technology, chisheu@yahoo.com

#### Chiun-Chieh Hsu

National Taiwan University of Science and Technology, cchsu@mail.ntust.edu.tw

#### Yu-Chun Chu

National Taiwan University of Science and Technology, D9509104@mail.ntust.edu.tw

Rotator graphs, first proposed by Corbett, have been studied in recent years. Rotator graphs were modified by adding generation functions to make all edges bi-directional, which is called bi-rotator graphs. Bi-rotator graphs, like all Cayley graphs, possess rich structural properties, such as symmetry, low diameter, and recursive construction. A bi-rotator graph of scale n, denoted as n-BR, contains n! nodes and the degree of each node is 2n-3. Spanning trees are said to be independent if a directed edge can only be consisted in one tree, where multiple independent spanning trees can be applied to concurrent message transmission and fault tolerance broadcasting. The algorithm proposed in this paper adopts recursive construction for finding all independent spanning trees. With the increasing of the scale, two edges are added to each node. Based on the 4-BR, we can construct all independent spanning trees of the 5-BR using the rich structural properties and the added neighbors. In a similar way, we can construct independent spanning trees of the n-BR based on the spanning tree construction of the (n-1)-BR. Applying the properties of symmetry and recursive construction, the paper proposes an efficient algorithm of constructing independent spanning trees for a bi-rotator graph. The proposed method constructs 2n-3 independent spanning trees rooted at a designated node for an n-BR, where the number of the spanning tree is proved to be maximum.

# A dual-ascent-based branch-and-bound framework for the prize-collecting Steiner tree and related problems

Martin Luipersbeck

University of Vienna, martin.luipersbeck@univie.ac.at

Markus Leitner

University of Vienna, markus.leitner@univie.ac.at

#### Ivana Ljubic

ESSEC Business School of Paris, ivana.ljubic@essec.edu

#### Markus Sinnl

University of Vienna, markus.sinnl@univie.ac.at

A branch-and-bound framework based on a dual ascent algorithm for the directed (asymmetric) prize-collecting Steiner tree problem (APCSTP) is presented. The framework is computationally explored in combination with various other algorithmic ingredients, including reduction tests, branching strategies and primal heuristics.

Given a graph G = (V, A) with edge weights  $c : A \to \mathbb{R}_{\geq 0}$ , node weights  $p : V \to \mathbb{R}_{\geq 0}$ , root node  $r \in V$  and terminals  $T \subset V$ , the goal of the APCSTP is to find an arborescence  $S = (S_V, S_A) \subset G$  rooted at r that spans T and such that  $\sum_{(i,j)\in S_A} c_{ij} + \sum_{i\notin S_V} p_i$  is minimized.

This problem definition is very general, as many network design problems can be transformed to it, including the prize-collecting Steiner tree problem on an undirected graph (PCSTP), the maximum-weight connected subgraph problem (MWCS), the node-weighted Steiner tree problem (NWSTP), and the Steiner tree problem (STP). These problems are well-known and cover a broad range of relevant applications.

The presented dual ascent algorithm is a fast procedure for obtaining a valid lower bound by computing a heuristic solution to the dual of a cutbased ILP formulation. Our algorithm can be seen as a generalization of the Goemans-Williamson primal-dual algorithm for the PCSTP [1] and the dual ascent algorithm for the STP proposed by Wong [2]. The bounds produced by the latter are empirically very tight, and the algorithm has been applied as building block in the design of successful algorithms for the STP [3,4].

Since to the best of our knowledge reduction tests have only been proposed for the PCSTP in its undirected form, we formulate new bound-based and alternative-based reduction tests for the APCSTP.

Our computational results show that the implemented framework manages to solve to optimality most instances from the PCSTP literature, and frequently outperforms recently proposed exact and heuristic state-of-the-art algorithms (see, e.g., [3]). Especially the results on large-scale graphs show the significance of the presented methods. Similar results are reported for the MWCS and NWSTP.

#### References

- 1 Goemans, M.X., Williamson, D.P.: A general approximation technique for constrained forest problems. SIAM Journal on Computing 24(2) (1995) 296–317
- 2 Wong, R.T.: A dual ascent approach for Steiner tree problems on a directed graph. Mathematical programming 28(3) (1984) 271–287
- 3 Fischetti, M., Leitner, M., Ljubic, I., Luipersbeck, M., Monaci, M., Resch, M., Salvagnin, D., Sinnl, M.: Thinning out Steiner trees: a nodebased model for uniform edge costs. In: Workshop of the 11th DIMACS Implementation Challenge. (2014)
- 4 Polzin, T., Daneshmand, S.V.: Improved algorithms for the Steiner problem in networks. Discrete Applied Mathematics 112(1) (2001) 263–300

### An algorithm for finding a representation of a subtree distance

Kazutoshi Ando

Faculty of Engineering, Shizuoka University, ando.kazutoshi@shizuoka.ac.jp

#### Koki Sato

Faculty of Engineering, Shizuoka University

We call a mapping  $d : X \times X \to \mathbb{R}_+$  a dissimilarity mapping on X if for each  $x, y \in X$  we have d(x, y) = d(y, x) and d(x, x) = 0. A dissimilarity mapping d on X is a tree distance if there exists a weighted tree T with its vertex set containing X such that  $d(x, y) = d_T(x, y)$  for all  $x, y \in X$ , where  $d_T(x, y)$  denotes the length of the unique path in T connecting x and y. Buneman (1974) characterized the tree distances by a condition called the four point condition. Also, there exists an  $O(n^3)$  algorithm (Saitou et al. (1987)) for finding a representation of a given tree distance.

Generalizing the concept of tree distance, Hirai (2006) introduced the concept of subtree distance. A dissimilarity mapping d on X is called a subtree distance if there exists a weighted tree T and a family  $(T_x | x \in X)$  of subtrees of T indexed by the elements in X such that  $d(x, y) = d_T(T_x, T_y)$ , where  $d_T(T_x, T_y)$  is the distance between  $T_x$  and  $T_y$  in T. Hirai (2006) gave a characterization of subtree distances which corresponds to the four point condition (Buneman (1974)) for the tree distances. Using this characterization, we can decide whether given matrix is a subtree distance or not in time  $O(n^4)$ . However, the existence of a polynomial time algorithm for finding a tree and subtrees representing a given subtree distance has been an open question. In this paper, we show an  $O(n^3)$  time algorithm that find a tree and subtrees representing a given dissimilarity mapping. By using this algorithm, one can decide in  $O(n^3)$  time whether or not a given dissimilarity mapping is a subtree distance.

#### References

 P. Buneman. A note on metric properties of trees. Journal of Combinatorial Theory, Series B 17 (1974) 48-50.

- [2] H. Hirai:. Characterization of the distance between subtrees of a tree by the associated tight span. Annals of Combinatorics **10** (2006) 111-128.
- [3] N. Saitou and M. Nei. The neighbor-joining method: a new method for reconstructing phylogenetic trees. Molecular Biology and Evolution 4 (1987) 405-425.

### Shared multicast trees in ad hoc wireless networks

#### Marika Ivanova

Institute of Informatics, University of Bergen, ivanova.marika@gmail.com

This paper addresses a problem of shared multicast trees (SMT), which extends a recently studied problem of shared broadcast trees (SBT). In SBT, a common optimal tree for a given set of nodes allowing broadcasting from any node to the rest of the group is searched. In SMT, also nodes that neither initiate any transmission, nor act as destinations are considered. Their purpose is exclusively to relay messages between nodes. The optimization criterion is to minimize the energy consumption. The present work introduces this generalization and devises solution methods. We model the problem as an integer linear program (ILP), in order to compute the exact solution. However, the size of instances solvable by ILP is significantly limited. Therefore, we also focus on inexact methods allowing us to process larger instances. We design a fast greedy method and compare its performance with adaptations of algorithms solving related problems. Numerical experiments reveal that the presented greedy method produces trees of lower energy than alternative approaches, and the solutions are close to the optimum.

# The maximum matrix contraction problem

#### Dimitri Watel

ENSIIE / CEDRIC CNAM, dimitri.watel@ensiie.fr

#### **Pierre-Louis Poirion**

ENSILE / CEDRIC CNAM, pierre-louis.poirion@ensta-paristech.fr

In this paper, we introduce the Maximum Matrix Contraction problem, where we aim to contract as much as possible a binary matrix in order to maximize its density. We study the complexity and the polynomial approximability of the problem. Especially, we prove this problem to be NP-Complete and that every algorithm solving this problem is at most an  $\sqrt{n}$ -approximation algorithm. We then focus on efficients algorithm to solve the problem: a linear program and three heuristics.

# Approximation algorithms for the k-hop connected dominating set problem

Rafael S. Coelho

University of São Paulo, rafaelsantoscoelho@gmail.com

#### Yoshiko Wakabayashi

University of São Paulo, yw@ime.usp.br

Let G be a connected graph and k be a positive integer. A vertex subset D of G is a k-hop connected dominating set (or k-CDS, for short) if D induces a connected subgraph of G and if every vertex of G is within distance at most k from some vertex in D. In this work, we tackle the problem (referred to hereafter as MINk-CDS) of finding a k-CDS of minimum cardinality of a given graph G. For every fixed  $k \geq 2$ , we present a  $k(1+\epsilon)(1+\ln(\Delta(G^k)-1))$ approximation algorithm for the problem, for every fixed  $0 < \epsilon \leq 1$ , thus improving on the best one known so far, which has a factor of  $2k(1+H(\Delta(G^k)))$ , where  $\Delta(G^k)$  is the maximum degree of the kth power of the input graph G and H(n) denotes the n-th harmonic number. Furthermore, we also give two approximation algorithms for the weighted version of MINk-CDS restricted to a special class of graphs that includes bounded treewidth graphs, bounded degree graphs and chordal graphs. To the best of our knowledge, for  $k \geq 2$ , these approximation algorithms for MINk-WCDS are the first ones to appear in the literature.

# Approximating interval selection on unrelated machines with unit-length intervals and cores

Matúš Mihalák

Maastricht University, matus.mihalak@maastrichtuniversity.nl

Katerina Bohmova ETH Zurich, asitak.kat@gmail.com

Enrico Kravina ETH Zurich, enrico.kravina@gmail.com

We consider a scheduling problem with machine dependent intervals, where each job consists of m fixed intervals, one on each of the m machines. To schedule a job, exactly one of the m intervals needs to be selected, making the corresponding machine busy for the time period equal to the selected interval. The objective is to schedule a maximum number of jobs such that no two selected intervals from the same machine overlap. This problem is NPhard and admits a deterministic 1/2-approximation. The problem remains NP-hard even if all intervals have unit length, and all m intervals of any job have a common intersection. We study this special case and show that it is APX-hard, and design a 501/1000-approximation algorithm.

# Approximability and exact resolution of the multidimensional binary vector assignment problem

Marin Bougeret LIRMM, marin.bougeret@lirmm.fr

Guillerme Duvillié LIRMM, guillerme.duvillie@lirmm.fr

Rodolphe Giroudeau *LIRMM*, rgirou@lirmm.fr

In this paper we consider the multidimensional binary vector assignment problem. An input of this problem is defined by m disjoint sets  $V^1, V^2, \ldots, V^m$ , each composed of n binary vectors of size p. An output is a set of n disjoint *m*-tuples of vectors, where each *m*-tuple is obtained by picking one vector from each set  $V^i$ . To each *m*-tuple we associate a *p* dimensional vector by applying the bit-wise AND operation on the m vectors of the tuple. The objective is to minimize the total number of zeros in these n vectors. We denote this problem by min  $\sum 0$ , and the restriction of this problem where every vector has at most c zeros by  $(\min \sum 0)_{\#0 < 1}$ .  $(\min \sum 0)_{\#0 < 2}$  was only known to be APX-complete, even for  $m = \overline{3} [1]$ . We show that, assuming the unique games conjecture, it is NP-hard to  $(n - \varepsilon)$ -approximate  $(\min \sum 0)_{\#0 < 1}$  for any fixed n and  $\varepsilon$ . This result is tight as any solution is a n-approximation. We also prove without assuming UGC that  $(\min \sum 0)_{\#0 \le 1}$  is APX-complete even for n = 2, and we provide an example of n - f(n, m)-approximation algorithm for min  $\sum 0$ . Finally, we show that  $(\min \sum 0)_{\#0 < 1}$  is polynomial for fixed m (which cannot be extended to  $(\min \sum 0)_{\#0<1}$  according to [1]).

#### References

[1] T. Dokka, Y. Crama and F.C. Spieksma.: Multidimensional vector assignment problems, Discrete Optimization, 14:111-125, 2014.

# On the finite optimal convergence of logic-based Benders' Decomposition in solving 0-1 min-max regret optimization problems with interval costs

Lucas Assunção

Universidade Federal de Minas Gerais, lucas.suncao88@gmail.com

Andréa Cynthia Santos

Université de Technologie de Troyes, andrea.duhamel@utt.fr

Thiago F. Noronha UFMG, tfn@dcc.ufmg.br

Rafael Andrade

Federal University of Ceará, rca@lia.ufc.br

This paper addresses a class of problems under interval data uncertainty composed of min-max regret versions of classical 0-1 optimization problems with interval costs. We refer to them as interval 0-1 min-max regret problems. The state-of-the-art exact algorithms for this class of problems work by solving a corresponding mixed integer linear programming formulation in a Benders' decomposition fashion. Each of the possibly exponentially many Benders? cuts is separated on the fly through the resolution of an instance of the classical 0-1 optimization problem counterpart. Since these separation subproblems may be NP- hard, not all of them can be modeled by means of linear programming, unless P = NP. In these cases, the convergence of the aforementioned algorithms are not guaranteed in a straightforward manner. In fact, to the best of our knowledge, their finite convergence has not been explicitly proved for any interval 0-1 min-max regret problem. In this work, we formally describe these algorithms through the definition of a logic-based Benders' decomposition framework and prove their convergence to an optimal solution in a finite number of iterations. As this framework is applicable to any interval 0-1 min-max regret problem, its finite optimal convergence

also holds in the cases where the separation subproblems are NP-hard.

## A review of algorithmic enhancements for Benders decomposition

Halil Sen

University of Bordeaux, halil.sen@inria.fr

**Boris Detienne** 

University of Bordeaux, boris.detienne@math.u-bordeaux1.fr

Ruslan Sadykov University of Bordeaux, ruslan.sadykov@inria.fr

#### Francois Vanderbeck

University of Bordeaux, francois.vanderbeck@math.u-bordeaux1.fr

In Benders decomposition approach to mixed integer programs, the optimization is carried in two stages: key first-stage decision variables are optimized using a polyhedral approximation of the full-blown problem projection, then a separation problem expressed in the second-stage variables is solved to check if the current first-stage solution is truly feasible, and otherwise, it produces a violated inequality. Such cutting-plane algorithms suffer from several drawbacks and may have very bad convergence rates. We review the battery of approaches that have been proposed in the literature to address these drawbacks and to speed-up the algorithm. Our contribution consists in explaining these techniques in simple terms and unified notations, showing that in several cases, different proposals of the literature boil down to the same key ideas. We classify methods into specific initialization mode, stabilization techniques, strategies to select the separation point, and cut generation strategies. Where available, we highlight numerical benchmarks that have resulted from such enhancements.

### General disjunction branching based on objective function improvement

Stephen Maher

Zuse Institute Berlin, maher@zib.de

**Gregor Hendel** Zuse Institute Berlin, hendel@zib.de

#### Yuji Shinano Zuse Institute Berlin, shinano@zib.de

Many components of mixed integer programming solvers partition the feasible region to help identify improved primal and dual bounds. Partitioning is used in heuristic and exact methods with an aim to exploit the general expectation that the resulting restricted problems are easier to solve than the original. Heuristically mixed integer programming solvers exploit the effects of partitioning through the use of large-neighbourhood search heuristics. Exactly branching rules are used to progressively divide a problem into smaller subproblems to achieve feasibility and optimality. The continued rise of large-scale parallel computing environments provides the ideal framework to utilise the concept of partitioning by concurrently searching unconventionally defined subproblems.

The branching scheme investigated in this talk integrates concepts from proximity search [1], local branching [2] and branching on general disjunctions [3]. Each of these approaches have been investigated in isolation and have been shown to significantly improve the performance of mixed integer programming solvers. While the approaches are complementary, their combined use in the form of an exact branching scheme has not been adequately examined. The combined used of the developed partitioning approach and parallel computing will aim to improve solving performance on difficult mixed integer programming problems. The presented branching scheme constructs partitions of the feasible region using general disjunctions.

Consider a minimisation problems with objective function  $c^T x$  that is always integer. Given an incumbent and LP relaxation solution with objective function values  $z^*$  and  $z^{LP}$  respectively, the following disjunction is formed:

$$\left(\left\lceil z^{LP}\right\rceil \le c^T x \le z^* - 3\right) \lor \left(z^* - 2 \le c^T x \le z^* - 1\right)$$

The right partition is reminiscent of local branching [2] and the left draws upon concepts of using general disjunctions to quickly find feasible solutions [3]. Further, techniques from proximity search [1] are applied through the use of the objective function to form the partition.

The proposed partitioning approach will be applied in sequential and parallel settings. The impact of the developed branching scheme will be assessed using the mixed integer programming solver SCIP. A comparison will be made by comparing the number of processed nodes, the primal integral and solving time.

#### References

- 1 M. Fischetti and M. Monaci. Proximity search for 0-1 mixed-integer convex programming. Journal of Heuristics, 20(6):709–731, 2014.
- 2 M. Fischetti and A. Lodi. Local branching. Mathematical Programming, 98:23–47, 2003.
- 3 H. Mahmoud and J. W. Chinneck. Achieving MILP feasibility quickly using general disjunctions. Computers & Operations Research, 40(8):2094–2102, 2013.

# A decomposition approach for single allocation hub location problems with multiple capacity levels

#### Borzou Rostami

Technische Universität Dortmund, brostami@mathematik.tu-dortmund.de

#### Christopher Strothmann Technische Universität Dortmund,

christopher.strothmann@tu-dortmund.de

#### Christoph Buchheim Technische Universität Dortmund, christoph.buchheim@math.tu-dortmund.de

In this paper we consider an extended version of the classical capacitated single allocation hub location problem in which the size of the hubs must be chosen from a finite and discrete set of allowable capacities. We develop a Lagrangian relaxation approach that exploits the problem structure and decomposes the problem into a set of smaller subproblems that can be solved efficiently. Upper bounds are derived by Lagrangean heuristics followed by a local search method. Moreover, we propose some reduction tests that allow us to decrease the size of the problem. Our computational experiments on some challenging benchmark instances from literature show the advantage of the decomposition approach over commercial solvers.

# Exact solution methods for the k-item quadratic knapsack problem

Lucas Létocart

Universite Paris 13, Sorbonne Paris Cite, LIPN, CNRS, (UMR 7030), lucas.letocart@lipn.univ-paris13.fr

#### Angelika Wiegele

Alpen-Adria-Universität Klagenfurt, angelika.wiegele@aau.at

The purpose of this paper is to solve the 0-1 k-item quadratic knapsack problem that can be formulated as follows.

$$(kQKP) \begin{cases} \max & \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{i} x_{j} \\ \text{s.t.} & \sum_{j=1}^{n} a_{j} x_{j} \leq b \\ & \sum_{j=1}^{n} x_{j} = k \\ & x_{j} \in \{0,1\} \quad j = 1, \dots, n \end{cases}$$
(1)

(kQKP) maximizes a quadratic function subject to a budget constraint (1), as in knapsack problems, and a cardinality constraint (2), as in k-densest subgraph problems. As it includes two classical NP-hard subproblems, (kQKP)is NP-hard.

We propose an exact solution method based on semidefinite relaxations. The "standard procedure" for obtaining a semidefinite relaxation of a quadratic 0-1 problem yields a semidefinite optimization problem without strictly feasible points. However, the presence of interior points ensures strong duality and thus having an interior is essential for solving semidefinite problems. We give a recipe how to transform the problem without strictly feasible points into one having an interior. The resulting relaxation has another nice property, namely that all constraint matrices have rank one. This reduces the number of matrix multiplications when using interior point methods and we can solve the semidefinite problem efficiently.

In order to tighten the bounds, we strengthen the relaxation by polyhedral constraints, namely the so-called triangle inequalities. The strengthened semidefinite problem is intractable for interior point methods; instead we design a bundle method (as introduced in Fischer at al. (2006)) in order to obtain approximate solutions that are guaranteed to be upper bounds on (kQKP).

This upper bound is then used together with heuristics for obtaining lower bounds from Létocart et. al. (2014) inside a branch-and-bound framework.

We briefly review other exact solution methods and compare all these approaches by experimenting with instances of various sizes and densities.

#### References

- Ilse Fischer, Gerald Gruber, Franz Rendl, and Renata Sotirov. Computational experience with a bundle approach for semidefinite cutting plane relaxations of Max-Cut and equipartition. Math. Programming, 105 (2-3, Ser. B): 451–469, 2006.
- [2] Lucas Létocart, Marie-Christine Plateau, and Gérard Plateau. An efficient hybrid heuristic method for the 0-1 exact k-item quadratic knapsack problem. Pesquisa Operacional, 34(1): 49–72, 2014.

## On interdiction problems over independence systems

Markus Sinnl

University of Vienna, markus.sinnl@univie.ac.at

Matteo Fischetti University of Padova, fisch@dei.unipd.it

Michele Monaci University of Padova, michele.monaci@unipd.it

#### Ivana Ljubic

ESSEC Business School of Paris, ivana.ljubic@univie.ac.at

In many real world optimization problems, there is no single decisionmaker, but multiple decision parties, which may have different, conflicting goals. These problems can be modeled using multilevel optimization, in which decisions are made hierarchically, at multiple levels. A decision made at a certain level is influenced by the decisions of previous levels, and it also influences the outcome at all remaining levels.

In this work, we focus on interdiction problems, which are a special case of bilevel optimization. From the game-theoretical perspective, we will say that there are two players, a leader and a follower. The follower solves a maximization problem over a set of items. The leader, who makes the decision at the first level, wants to interdict a subset of the follower items in such a way, that the objective of the follower is minimized. If an item is interdicted, the follower cannot use it. The leader has an interdiction budget constraint (and potentially some additional constraints) and thus cannot interdict all items. The follower problem studied in our case is maximization of a linear function over an independence system, which means, if item set I is feasible for the follower problem for a given interdiction of the leader, then  $I' \subset I$ is also feasible. Note that the (multi-dimensional) knapsack-problem falls into this category. The problem is known to be  $\Sigma_2^p$ -complete even for the knapsack-problem as follower problem [1,2].

We present a decomposition-based integer programming approach. For a given subset of interdicted items (representing a solution of the master problem), an optimal solution of the follower is found and optimality cuts are produced. Unlike Benders decomposition, using duality theory does not provide valid optimality cuts. Instead, optimality cuts are obtained by exploiting the independence system structure of the follower. We show how to lift the obtained optimality cuts and describe further valid inequalities.

A branch-and-cut algorithm based on our decomposition approach is implemented. The algorithm also contains preprocessing and separation heuristics. A computational study on interdiction knapsack instances from the literature is done to assess the computational efficiency of our approach against previous approaches [1,2]. We also provide computations for multidimensional knapsack followers.

#### References

- DeNegre, S.: Interdiction and discrete bilevel linear programming. PhD thesis (2011)
- [2] Caprara, A., Carvalho, M., Lodi, A., Woeginger, G.: Bilevel knapsack with interdiction constraints. INFORMS Journal on Computing (to appear)

### Discrete conditional value-at-risk

Carlo Filippi

Department of Quantitative Methods, University of Brescia, carlo.filippi@unibs.it

Wlodzimierz Ogryczak

Warsaw University of Technology, Institute of Control and Computation Engineering, ogryczak@ia.pw.edu.pl

#### M. Grazia Speranza

Department of Quantitative Methods, University of Brescia, speranza@eco.unibs.it

The trade-off between risk and return in financial investments has been deeply investigated, starting from Markowitz model, where risk was measured with the variance of the portfolio rate of return. In general, the risk of an investment is associated with the uncertainty on the future return. Several measures have been proposed in finance to measure the uncertainty of the portfolio rate of return. The Value-at-Risk (VaR) and the Conditional Valueat-Risk (CVaR) have recently become very popular concepts. Whereas the VaR has some undesirable characteristics, the CVaR has all the desirable characteristics needed to be defined a coherent risk measure. The CVaR is widely used in finance to measure and compare portfolios. The concept of CVaR is built upon the assumption that the rates of return of the assets follow a known probability distribution.

The control of uncertainty is vital in finance but is extremely important in many other areas. Often parameters of an optimization model are not known with certainty and deterministic models are built upon estimations. The coefficients of the objective function are usually recognized as the most relevant parameters. Several efforts are being made to model uncertainty. One of the most commonly adopted techniques is scenario generation. Let us assume that S scenarios for the coefficients of the objective function of a mixed integer linear programming (MILP) model are generated. All scenarios are equally probable and we aim at being protected against a certain number, say k, of the worst scenarios, that is against a fraction k/S of the worst possible values of the objective function. In a minimization problem, a sensible objective becomes the minimization of the average value of the objective function in the k worst scenarios. We call this measure Discrete CVaR (DCVaR).

In this paper, we focus on optimization problems that can be formulated as Mixed Integer Linear Programming (MILP) models. We consider two different settings where one may find the optimization of the average or total payoff and the minimax/maximin approach unsatisfactory. In the first setting, the coefficients of the objective function are uncertain and uncertainty is described through a finite set of possible scenarios. In the second setting all input data is known but the variability of the payoffs (coefficients) in the objective function is a concern for the decision maker. The main goal of this paper is to unify the two above settings and to show the broad applicability of the DCVaR concept. We will show that adopting the DVCaR leads to another mixed integer linear program. We will also show that, if we interpret the individual payoffs as the only, equally probable, values of a discrete random variable, the DCVaR is equivalent to the classical CVaR. We will also show that the DCVaR has several desirable properties. We complement the theoretical part with two examples of application of the DCVaR, namely the p-center/p-median problem, as an example of variability, and the multi-dimensional knapsack, as an example of uncertainty.
# MIP formulations for a rich real-world lot-sizing problem with setup carryover

Filippo Focacci DecisionBrain, sbianchi@fceia.unr.edu.ar

Fabio Furini Paris Dauphine, fabio.furini@dauphine.fr

Virginie Gabrel Paris Dauphine, virginie.gabrel@dauphine.fr

Daniel Godard DecisionBrain, daniel.godard@decisionbrain.com

#### Xueying Shen DecisionBrain, Paris Dauphine, sylvia.shen@decisionbrain.com

A rich lot-sizing problem is studied in this manuscript which comes from a real-world application. Our new lot-sizing problem combines several features, i.e., parallel machines, production time windows, backlogging, lost sale and setup carryover. Three mixed integer programming formulations are proposed. We theoretically and computationally compare these different formulations, testing them on real-world and randomly generated instances. Our study is the first step for efficiently tackling and solving this challenging real-world lot-sizing problem.

## Two-level supply chain coordination under complete or asymmetric information

Siao-Leu Phouratsamay UPMC, siao-leu.phouratsamay@lip6.fr

Safia Kedad-Sidhoum UPMC, safia.kedad-sidhoum@lip6.fr

Fanny Pascual UPMC, fanny.pascual@lip6.fr

We consider a supply chain with two actors: a supplier and a retailer. The retailer has to satisfy a demand for a single item over a finite planning horizon of discrete periods. In order to satisfy the demand, the retailer has to determine a replenishment plan, i.e. he has to decide when to order and how many units to order. The supplier has to determine a production plan in order to satisfy the retailer's replenishment plan. Order units induce a fixed ordering cost and a unit ordering cost for both actors. Carrying units in the inventory induce a unit holding cost for both actors. We are interesting in the coordination of the lot-sizing decisions of the actors so as to minimize their cost.

Without cooperation, the retailer will compute his optimal replenishment plan, and will impose it to the supplier. In this case, the cost of the supplier for satisfying this plan can be very high. However, if the actors decide to collaborate, the supplier can lower his cost by proposing a new replenishment plan to the retailer. Our aim is to compute a replenishment plan for the retailer that is interesting, and that minimizes the supplier's cost. The cost of the replenishment plan proposed by the supplier can be greater than the retailer's optimal cost. In order to persuade the retailer to accept the new replenishment plan, a side payment between the actors is allowed, the supplier can assume part of the retailer's cost.

In this work, we consider different supply chain collaboration scenarios: no side payment is allowed, a side payment on the retailer's holding cost is allowed, and a side payment on the retailer's ordering and holding costs is

#### allowed.

In order to optimize his cost, the supplier proposes to the retailer a contract composed of a replenishment plan and a side payment if there is one. The retailer may accept the contract if its associated cost does not exceed the cost that the retailer would have by choosing himself his optimal replenishment plan. Then, the supplier needs to know the retailer's costs. However, in practice, the supplier is not always able to get a complete information about the retailer's costs. In this case, the supplier will have to take this information asymmetry into account to design the contracts.

In this study, we present different models aiming at optimizing the supplier's total cost under the following hypothesis: either the supplier has complete information about the retailer's costs, or there is asymmetric information between the actors. In both cases we consider the case where side payments are allowed or not. Solving methods for polynomial cases will be presented as well as an experimental analysis to show the relevance of the studied models. When the actors collaborate, the experimental analysis shows that the supplier's cost can be lowered under both hypothesis. This work is one of the first dealing with coordination mechanisms for multi period dynamic lot-sizing problems under asymmetric information.

# On robust lot sizing problems with storage deterioration, with applications to heat and power cogeneration

Stefano Coniglio

University of Southampton, stefano.coniglio@gmail.com

Arie Koster

RWTH University Aachen, koster@math2.rwth-aachen.de

#### Nils Spiekermann

RWTH University Aachen, spiekermann@math2.rwth-aachen.de

We consider a variant of the single item lot sizing problem where the product, when stored, suffers from a proportional loss, and in which the product demand is affected by uncertainty. This setting is particularly relevant in the energy sector, where the demands must be satisfied in a timely manner and storage losses are, often, unavoidable. We propose a two-stage robust optimization approach to tackle the problem with second stage storage variables. We first show that, in the case of uncertain demands, the robust problem can be solved as an instance of the deterministic one. We then address an application of robust lot sizing arising in the context of heat and power cogeneration and show that, even in this case, we can solve the problem as an instance of the deterministic lot sizing problem. Computational experiments are reported and illustrated.

### The closest string problem with 4-string is enough for its NP-hardness

Omar Latorre

*IComp/UFAM*, omarlatorre@icomp.ufam.edu.br

Rosiane de Freitas IComp/UFAM, rosiane@icomp.ufam.edu.br

We proved the k closest string problem (k-CSP) with k greater than or equal to four is NP-hard. The general CSP is NP-complete, so we given a polynomial time reduction from 3-SAT to the 4-CSP and thus, as the problem for 3-string was already indicated to be a polynomial problem, we prove that 4-CSP constitutes the smallest CSP decision problem belonging to the NPcomplete class, and the full complexity dichotomy for this problem is given. Besides, we also give the proof of correctness for efficient algorithms in linear time for the Closest String Problem with 2,3-strings.

### Sum-of-Squares rank upper bounds for matching problems

Samuli Leppänen IDSIA, samuli@idsia.ch

Adam Kurpisz Wroclaw University of Technology, adam.kurpisz@pwr.wroc.pl

> Monaldo Mastrolilli IDSIA, monaldo@idsia.ch

The matching problem is one of the most studied combinatorial optimization problems in the context of extended formulations and convex relaxations. In this paper we provide upper bounds for the rank of the Sum-of-Squares (SoS)/Lasserre hierarchy for a family of matching problems. In particular, we show that when the problem formulation is strengthened by incorporating the objective function in the constraints, the hierarchy requires at most  $\lceil \frac{k}{2} \rceil$ rounds to refute the existence of a perfect matching in an odd clique of size 2k + 1.

### Optimum solution of the closest string problem via rank distance

**Claudio Arbib** 

Universit degli Studi dellAquila, claudio.arbib@univaq.it

Giovanni Felici

Istituto di Analisi dei Sistemi e Informatica, Consiglio Nazionale delle Ricerche, giovanni.felici@gmail.com

Mara Servilio

Istituto di Analisi dei Sistemi e Informatica, Consiglio Nazionale delle Ricerche, mservilio@gmail.com

#### Paolo Ventura

Istituto di Analisi dei Sistemi e Informatica, Consiglio Nazionale delle Ricerche, paolo.ventura@iasi.cnr.it

The Closest String Problem (CSP) calls for finding an n-dimensional string that minimizes, for a given metric, its maximum distance from a given target set of m strings of the same dimension. Such a string can thus be considered a center of the target set. Under some metrics, such as the Hamming distance, integer linear programming (ILP) proved to be able to find optimal solutions to relatively large CSPs, thus providing a basis for successful employment in math-heuristics. In computational biology, however, Hamming distance seems to be outperformed by more complex metrics that take into account insertions and deletions - the most established one being the Levenshtein distance, also referred to as edit distance. With edit distance, however, it does not appear easy to formulate the CSP as ILP. Recent research has demonstrated that another metric, called the rank distance, can provide interesting results from both computational and experimental standpoints when applied to genomic sequences. An advantage of the rank distance is that, in this case, the CSP can easily be formulated as an integer program - precisely as a multi-weighted matching problem. In this way, optimal solutions can be found for moderate string sizes; otherwise, dual gap provides useful information on how far feasible solutions are from optimality. The model is quite versatile, as it also allows to seek for Pareto-optimal solutions with respect to different objective functions. Two examples that, among others, may result interesting in application are a) find a string that minimizes the weighted average rank (or Hamming) distance from the target set among all strings within a given rank distance from target set; b) explore the set of all centers at bounded distance from the target set. In this work we extend an existing ILP formulation of the CSP under rank distance, and use it in a computational experience on biological data in order to evaluate its viability. We also investigate a way to efficiently enforce the formulation by cutting planes. In fact, the model encompasses a set of multiple choice knapsack constraints, thus separation of fractional solutions by (lifted) cover inequalities can be formulated as a knapsack problem. It is then interesting to observe that, due to the special structure of the rank distance between two strings, the coefficients of the latter knapsack problem have polynomial size in n, and therefore cover separation can be done in polynomial time.

## Designing optimal charging station networks for electric car sharing systems

Georg Brandstätter

University of Vienna, georg.brandstaetter@univie.ac.at

Markus Leitner University of Vienna, markus.leitner@univie.ac.at

Ivana Ljubic ESSEC Business School, ivana.ljubic@essec.edu

Mario Ruthmair University of Vienna, mario.ruthmair@univie.ac.at

In recent years, car sharing has received increasing attention as a flexible, vet affordable mode of transportation within cities. Because of their high efficiency in urban settings, as well as their environmental friendliness, electric vehicles are excellent candidates for use within such systems where customers often rent cars only for short distances and periods of time. However, despite advances in battery technology, the range of electric vehicles is still low when compared to that of conventionally powered vehicles. Moreover, recharging them takes longer than refueling cars with internal combustion engines. Therefore, a network of charging stations must be built within the system's operational area, where cars can be parked and recharged between trips. Since constructing these stations is not only costly, but also requires a large amount of public space, the location and size of these stations must be chosen carefully in order to ensure that the system can still operate efficiently. We present several integer linear programming formulations for solving the problem of optimally placing charging stations within a car sharing system's operational area, as well as finding their optimal size. Using a set of expected trips as an estimation for customer demand, our objective is to maximize the profit of those trips that can be satisfied by the constructed stations. To improve flexibility, customers can pick up a car at any sufficiently close station, as well as return it to any station near their destination (subject to the availability of a car and free charging slot, respectively). A limit on the number of stations and charging slots that can be built is imposed by a budget constraint. We also compare the performance of these formulations on a set of benchmark instances, which consist of both artificial instances as well as ones based on real-world data.

### Energy network management of an oil refinery

#### Elif Mete

Koc University, emete14@ku.edu.tr

Energy is a critical element in refineries to operate their facilities, as it can significantly affect production costs as well. The effective management of the energy system in the refinery can improve the economic and environmental performance significantly. Tupras Izmit Refinery operates large and complex utilities and energy systems to satisfy its dynamic energy demand. Refining processes in Tupras use energy in the form of electric and steam. Electrical demands come from electrical plant devices or from the power required to drive compressors, pumps and fans in air coolers with electric motors. Steam is provided with different pressure levels to serve the demand arisen from different types of consumers such as the reaction system, heat exchange network, power for pumps, compressors with steam turbines, pipe line tracing etc. Tupras Izmit Refinery tends to fulfill the energy demand mainly from its utility plants. The design of the utility plants allows multiple operational configurations to secure the process continuity and flexibility. Additionally, the refinery is connected to national grids in order to buy or sell electricity depending on the amount of production and the prices of the electricity in "day ahead" and "daily" electricity market. The main objective of the project is developing a decision support tool to follow the complex energy network of the refinery and manage it by determining the optimum working criteria and combinations of the equipments. The project has mainly 3 parts: data management, optimization & modelling and interface & reporting. In the first part, it is aimed to determine the online operational status of each unit. The data are collected from different sources (process laboratory, SAP etc.) and filtered according to the operational rules. After validation of the data, energy demand of the system is calculated by the mass and energy balances. In the optimization part of the project, the problem is formulated as mixed integer linear programming model in GAMS optimization package and solved by CPLEX 12 solver. The model determines operational configuration of the system by minimizing the operational cost. In the final part, a user friendly interface is developed. The interface allows to design a process flow chart with 25 different equipments via drag-and-drop and to display the online plant data of equipments. The optimization report contains a detailed the comparison of the online plant data and the optimization result for each equipment. The developed decision support tool enables to optimize the complex energy network of the refinery. It has been run for the first time on 08.04.2015 and achieved up to \$1680 cost reduction per day. The effective management of the energy system does not only decrease the cost of production, but also decreases the CO2 emission due to the effective use of fossil fuels. Compare to the 2014 emission data, it could have been observed 4% decrease in the emission rate, if the tool had been run throughout 2014.

# Robust optimization of wiring in wind-farms: a robust steiner tree problem

Cédric Bentz CEDRIC, CNAM, cedric.bentz@cnam.fr

Marie-Christine Costa *CEDRIC, CNAM / ENSTA-Paristech,* marie-christine.costa@ensta-paristech.fr

Daniel Porumbel CEDRIC, CNAM, daniel.porumbel@cnam.fr

> Thomas Ridremont CNAM, thomas.ridremont@cnam.fr

In this paper, we study a network design problem arising when optimizing the cabling of a wind energy farm. There is a close relation between this problem and the minimum rooted Steiner tree problem in graphs. However, what makes our problem original is that we are looking for "robust" solutions, i.e. solutions that minimize the number of terminals disconnected from the root in the case of a breakdown on a cable. We first prove that the associated decision problem is NP-complete, even when the solutions are restricted to spanning trees. Then, we propose mathematical models to which we add some cuts in order to solve them with CPLEX. We test them on real data sets and compare the effects of the models on the structure of the solution trees.

# Author index

Öztop, Hande, 79 Sůcha, Přemysl, 114 Andrade, Rafael, 133 Assunção, Lucas, 133 Bianchi, Silvia, 64 Buchheim, Christoph, 18 Coniglio, Stefano, 148 D'Andreagiovanni, Fabio, 92 Faenza, Yuri, 35 Gleixner, Ambros, 92 Imaizumi, Jun, 122 Koeppe, Matthias, 37 Koster, Arie, 148 Kurpisz, Adam, 150 Leppänen, Samuli, 150 Letchford, Adam N., 66 Macchia, Marco, 36 Marzi, Francesca, 66 Mastrolilli, Monaldo, 150 Montenegro, Maribel, 18 Noronha, Thiago F., 133 Oki, Taihei, 109 Rossi, Fabrizio, 66

Santos, Andréa Cynthia, 133 Shiina, Takayuki, 122 Smriglio, Stefano , 66 Spiekermann, Nils, 148 Timmermans, Veerle , 106 Wiegele, Angelika , 18 Zhou, Yuan, 37 Zinder, Yakov, 119

Agra, Agostinho, 99 Alves, Cláudio, 71 Ando, Kazutoshi, 126 Aprile, Manuel , 35 Arbib, Claudio, 151 Archetti, Claudia, 50 Artigues, Christian, 121 Astorino, Annabella, 28 Avella, Pasquale, 83 Averbakh, Igor, 81

Bärmann, Andreas, 77 Baffier, Jean-François, 73 Barbato, Michele, 88 Beasley, John E., 47 Ben Abdelaziz, Fouad, 82
Ben-Ameur, Walid, 61
Bendotti, Pascale, 56
Bentz, Cédric, 157
Bianchi, Silvia, 96
Billaut, Jean-Charles, 121
Billionnet, Alain, 118
Boccia, Maurizio, 83
Bohmova, Katerina, 131
Boland, Natashia, 50
Bougeret, Marin, 132
Brandstätter, Georg, 153
Briant, Olivier, 33
Buchheim, Christoph, 138
Bukata, Libor, 114

Carrabs, Francesco, 23 Cerulli, Raffaele, 23, 24, 98 Cevallos, Alfonso, 35 Chen, Jingchao, 93 Cheref, Azeddine, 121 Chu, Yu-Chun, 123 Clautiaux, François, 71 Clautiaux, François, 71 Clautiaux, Francois, 113 Cornaz, Denis, 74 Costa, Marie-Christine, 118, 157 Cunha, Alexandre Salles, 47 Czibula, Oliver, 119 D'Ambrosio, Ciriaco, 23 Dahl, Geir, 99 De Boeck, Jérôme, 116 de Freitas, Rosiane, 149 De Maio, Annarita, 45 Dell'Amico, Mauro, 34 Derbel, Houda, 67 Detienne, Boris, 135 Diarrassouba, Ibrahima, 38, 95 Dias, Gustavo, 22 Dupin, Nicolas, 41 Duvillié, Guillerme, 132

Eliiyi, Uğur, 79 Escalante, Mariana, 64 Esmailpour, Mehran, 70

Fairbrother, Jamie, 101 Felici, Giovanni, 151 Filippi, Carlo, 143 Fiorini, Samuel, 36 Fischetti, Matteo, 141 Fisikopoulos, Vissarion, 36 Focacci, Filippo, 145 Fortz, Bernard, 58, 116 Fouilhoux, Pierre, 56 Frank, András, 12 Froger, Aurélien, 43 Furini, Fabio, 145 Gaar, Elisabeth, 62 Gabrel, Virginie, 145 Gaudioso, Manlio, 28 Gelareh, Shahin, 58 Gellermann, Thorsten, 77 Gendreau, Michel, 43 Gendron, Bernard, 26 Giroudeau, Rodolphe, 132 Glorieux, Antoine, 61 Godard, Daniel, 145 Golden, Bruce, 48 Gottschalk, Corinna, 100 Gouveia, Luis, 86, 90 Grappe, Roland, 74, 88 Gu, Hanyu, 119 Guillot, Jeremy, 113 Hanafi, Said, 71 Hanzálek, Zdeněk, 114 Harks, Tobias, 106 Hassan, Lucas, 41 Hassan, Mohammed-Albarra, 42 Haufmann, Torkel Andreas, 99 Hendel, Gregor, 136 Hirai, Hiroshi, 109 Hsu, Chiun-Chieh, 123 Iori, Manuel, 34

Ivanova, Marika, 128

Iwamasa, Yuni, 54 Jarboui, Bassem, 67 John, Maximilian, 21 Kacem, Imed, 42 Kaibel, Volker, 16 Kamoun, Manel, 67 Kandiller, Levent, 39 Karrenbauer, Andreas, 21 Kedad-Sidhoum, Safia, 146 Khalili, Majid, 70 Kijima, Shuji, 89 Kizilay, Damla, 29 Knust, Sigrid, 31, 85 Kramer, Raphael, 34 Kravina, Enrico, 131 Lüthen, Hendrik, 100 Létocart, Lucas, 139 Lacroix, Mathieu, 74, 88 Laganà, Demetrio, 45 Laporte, Gilbert, 98 Latorre, Omar, 149 Le Xuan, Thanh, 31 Lee, Cheng-Jhe, 123 Leitner, Markus, 55, 90, 124, 153 Lersteau, Charly, 24 Letchford, Adam, 14, 101, 111 Liberti, Leo, 22

Liers, Frauke, 77 Liguori, Pedro Henrique, 95 Ljubic, Ivana, 55, 124, 141, 153 Luipersbeck, Martin, 124 M.Osman, Izzeldin, 42 Macedo, Rita, 71 Magnouche, Youcef, 59 Maher, Stephen, 136 Mahey, Philippe, 20 Mahjoub, A. Ridha, 95 Mahjoub, Meriem, 38 Mahjoub, Ridha, 38 Mannino, Carlo, 83 Martin, Sébastien, 59 Martin, Sebastien, 42 Mateus, Geraldo Robson, 26 Mattia, Sara, 53 Mehta, Deepak, 51 Mendoza, Jorge E., 43 Merkert, Maximilian, 77 Mete, Elif, 155 Miglionico, Giovanna, 28 Mihalák, Matúš, 131 Minoux, Michel, 103 Mir, Hasan, 82 Miranda Bront, Juan Jose, 75 Monaci, Michele, 141 Morais, Vinícius, 26

Morito, Susumu, 122 Mosteiro, Agustin, 75 Naddef, Denis, 33 Nasini, Graciela, 64, 96 Neamatian Monemi, Rahimeh, 58 Neto, Jose, 61 Ngueveu, Sandra Ulrich, 121 Nguyen, Dang Phuong, 103 Nguyen, Thanh Hai, 103 Nguyen, Viet Hung, 103 Nishiyama, Hiroshi, 89 Noorizadegan, Mahdi, 107 O'Sullivan, Barry, 51 Ogryczak, Wlodzimierz, 143 Ozcan, Sel, 39 Ozturk, Cemalettin, 51 Parize, Rémi, 41 Pascual, Fanny, 146 Paulusma, Daniel, 94 Peis, Britta, 100 Pereira, Jorge, 81 Pesneau, Pierre, 86, 113 Phouratsamay, Siao-Leu, 146 Picouleau, Christophe, 94 Pinheiro, Sofia, 99 Pinson, Eric, 43 Poikonen, Stefan, 48, 68

Poirion, Pierre-Louis, 73, 118, 129 Porumbel, Daniel, 157 Pousa, Federico, 75 Quesada, Luis, 51 Rafik, Mahdaoui, 110 Raiconi, Andrea, 23, 24 Ravi, R., 11 Recalde, Diego, 105 Rendl, Franz, 62 Ridremont, Thomas, 157 Riedler, Martin, 55 Ries, Bernard, 94 Rossi, André, 24 Rostami, Borzou, 138 Rottner, Cécile, 56 Rousseau, Louis-Martin, 43 Ruthmair, Mario, 55, 86, 90, 153 S. Coelho, Rafael, 130 Sadykov, Ruslan, 135 Salles Da Cunha, Alexandre, 20 Santos, Daniel, 86 Sato, Koki, 126 Schneider, Oskar, 77 Seifi, Abbas, 107 Sen, Halil, 135 Servilio, Mara, 151 Sevaux, Marc, 24

Severín, Daniel, 105 Shen, Xueying, 145 Shinano, Yuji, 136 Silvestri, Selene, 98 Sinnl, Markus, 124, 141 Sirdey, Renaud, 103 Speranza, M. Grazia, 50, 143 Strothmann, Christopher, 138 Suppakitpaisarn, Vorapong, 73 Türsel Eliiyi, Deniz, 29, 39, 79 Takehi, Yuho, 122 Tayebi, Hamed, 70 Tolomei, Paola, 96 Torres, Luis Miguel, 96 Torres, Ramiro, 105 Vaca, Polo, 105 Valle, Cristiano Arbex, 47 Vanderbeck, Francois, 135 Vasilyev, Igor, 83 Ventura, Paolo, 151 Voge, Marie-Emilie, 71 Vu, Anh, 111 Wagler, Annegret, 64 Wakabayashi, Yoshiko, 130 Waldherr, Stefan, 85 Wang, Xingyin, 48 Watel, Dimitri, 129

Wiegele, Angelika, 139 Wierz, Andreas , 100 Wolfler Calvo, Roberto, 88

Yamashita, Masafumi, 89 Yamauchi, Yukiko, 89